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Analysis of acoustic fountain generated by ultrasonic plane wave for different water depth

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1. Introduction

Recently, ultrasonic atomization has been using in the various industrial fields such as nanoparticle separation, ethanol concentration, and medical drug delivery[1)]. Application of ultrasonic atomization is gradually increasing and it is attracting great interest in the various fields. There is need to control precisely the ultrasonic atomization according to the using purpose. Many studies have been reported on effects of driving frequency, droplet size, cavitation, and capillary waves in ultrasonic atomization[2,3)]. However, the analysis of acoustic fountain shape, directly related to the quantity of ultrasonic atomization, is not enough. In this study, acoustic fountain shape depending on water depth is calculated theoretically by using radiation pressure theory of nonlinear acoustics, and the calculation results are compared with the experimental one.

2. Theory

Figure 1 shows the acoustic fountain generated on liquid surface by ultrasound radiated from the ultrasonic transducer located at the depth d. When the height of fountain surface at the horizontal distance r from the transducer center is h, the forces acting on this point (r, h) are shown in **Fig. 2**. In this figure, p_L, σ, ρ, and ρ_a are the radiation pressure, the surface tension and the density of the liquid, and the density of the air, respectively. The radiation pressure, the surface tension, and the gravitational force are acting on a surface element with the length ds.

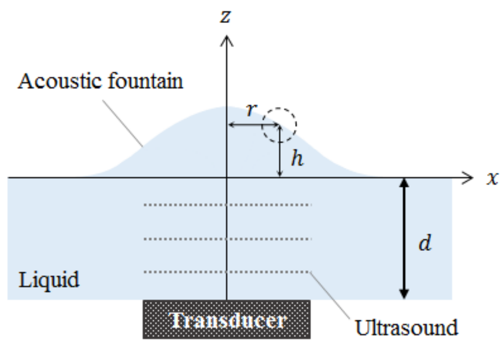

Fig. 1 Theoretical model of acoustic fountain.

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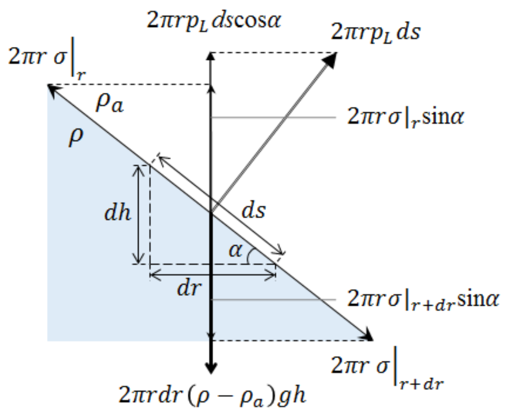

Fig. 2 Forces acting on boundary surface of fountain.

Considering that net force in the vertical direction is zero, Eq. (1) can be obtained.

$$\begin{aligned}&2\pi r p_L ds\cos\alpha + 2\pi r\sigma\sin\alpha\big|_r\\ &-2\pi r dr(\rho-\rho_a)gh - 2\pi r\sigma\sin\alpha\big|_{r+dr} = 0.\end{aligned} \tag{1}$$

From the trigonometric identity, Eq. (2) is obtained.

$$\sin\alpha = \frac{dh}{dr}\left[1+\left(\frac{dh}{dr}\right)^2\right]^{-1/2}. \tag{2}$$

And from Eqs. (1) and (2), the relationship between the factors related to the height of the fountain can be obtained as follows.

$$\begin{aligned}&\frac{1}{\sigma}\{p_L-(\rho-\rho_a)gh\}\left[1+\left(\frac{dh}{dr}\right)^2\right]^{3/2}\\ &=\frac{1}{r}\frac{dh}{dr}\left[1+\left(\frac{dh}{dr}\right)^2\right]+\frac{d^2h}{dr^2}.\end{aligned} \tag{3}$$

The radiation pressure p_L by acoustic nonlinear effect is given by[4)]

$$p_L = \frac{p_a^2}{2\rho c^2}+\frac{\rho(u_a\cdot u_a)}{2}. \tag{4}$$

Here p_a is the sound pressure radiated by the ultrasonic transducer, u_a is the particle velocity, and c is the sound velocity of the liquid. The sound pressure p_a from a piston source can be expressed by

$$p_a = \int_S\int_{S'} p_{mn}dS'dS. \tag{5}$$

Where S and S' are the area of the source plane and the field plane, respectively. When the field plane is located at $z=d$, p_{mn} at $P'(x'_m,\ y'_m,\ d)$ on the field plane is given by Eq. (6), which is a sound pressure element due to the source element at $P(x_n, y_n, 0)$ on the source plane.

$$p_{mn} = p_0 \frac{e^{-jk\sqrt{(x'_m - x_n)^2 + (y'_m - y_n)^2 + d^2}}}{\sqrt{(x'_m - x_n)^2 + (y'_m - y_n)^2 + d^2}} . \quad (6)$$

Here k is the wave number, and p_0 is the sound pressure at the piston surface.

3. Fountain shape measurement

To observe the fountain shape depending on water depth in real time, the measurement system was constructed, as shown in **Fig. 3**. The ultrasonic transducer of the diameter 1.87 cm radiates ultrasound of the resonant frequency 2 MHz from the bottom of the tank to form the fountain on the water surface. The changes of the fountain shape were recorded with a web camera (LifeCam Studio, Microsoft) for different depths of d. To make it easy to observe the fountain surface, black colored acoustic medium of 2 L, which was made of black water-color ink (LAMY, Germany) of 3 g, was used. At this time, the surface tension, the density, and the sound velocity of the solution are 64×10^{-3} N/m, 997 kg/m^3, and 1499 m/s, respectively.

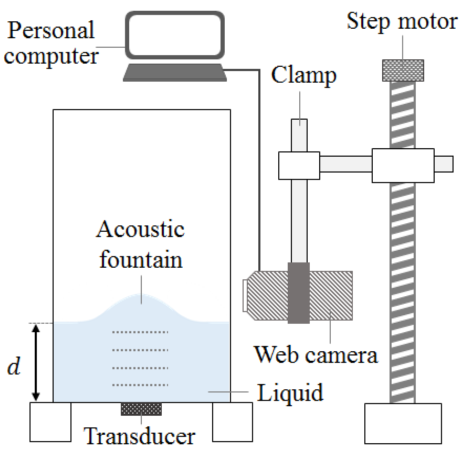

Fig. 3 Experimental system for observation.

4. Results

Figure 4 shows the images of the fountain for d=6, 7, 12, and 17 cm, when the electrical input power was 52.90 W. As the height of the fountain increases, as the water depth increases up to 7 cm. After then the height decreases with the water depth. When p_0=92 kPa, the theoretical calculation results for water depths of 10, 11, 16, and 21 cm are shown in **Fig. 5**. These results show that it is a similar tendency in the change of the fountain shape with water depth. Especially, the divergence in Fig. 5(b) is corresponding to the atomization in Fig. 4(b).

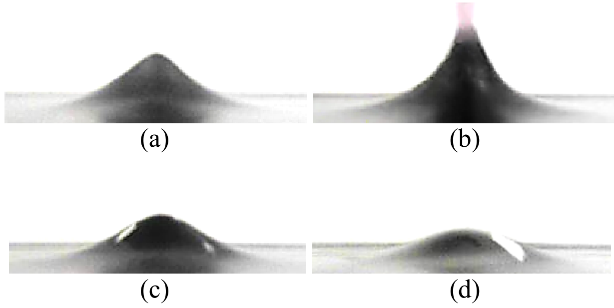

Fig. 4 Observed images of fountain depending on water depth, (a) 6 cm, (b) 7 cm, (c) 12 cm, and (d) 17 cm.

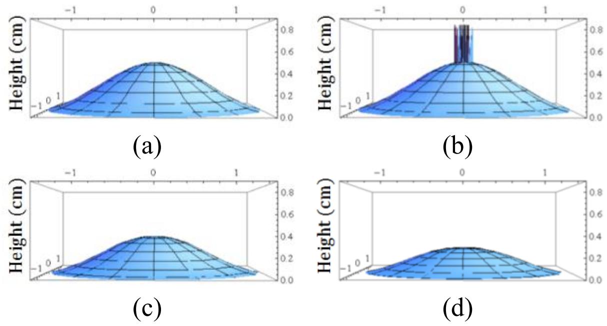

Fig. 5 Theoretical results of fountain depending on water depth, (a) 10 cm, (b) 11 cm, (c) 16 cm, and (d) 21 cm.

5. Summary

To analyze acoustic fountain generated by ultrasound, radiation pressure induced on the liquid surface by the sound pressure radiated from the ultrasonic transducer was calculated. The changes of the fountain shape depending on water depth were analyzed. In case of the ultrasonic transducer of 2 MHz, the maximum height of the fountain for ultrasonic atomization was observed to the depth of 7 cm. It was confirmed that the experimental results are consistent with the theoretical ones. From these results, the validation of the suggested analysis method in this study was verified.

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