2P1-3

FDTD Simulation of shear wave propagation in subcutaneous region

FDTD 法による皮下近傍での剪断波の伝搬シミュレーション

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1. Background

Elastography is one of diagnostic methods to evaluate human tissue noninvasively[1]. Dynamic elastography using optical coherence tomography (OCT) is expected to eastimate elastic property of small region near surface (< several mm) with a spatial resolution of less than 10 μm[2][3]. There are few studies about propagation characteristics of shear wave in subcutaneous region. In this study, Finite-difference time-domain（FDTD）method is applied to analyze propagation characteristics of shear wave.

2. FDTD simulation

Navie-stokes equations expressed in Eqs. (1)-(3) were used in 2-D FDTD simulation[4]. Here, v is particle velocity and σ is stress.

$$\rho\partial_t v_x = \partial_y \sigma_{xy} \tag{1}$$

$$\rho\partial_t v_y = \partial_x \sigma_{yx} \tag{2}$$

$$\partial_t \sigma_{xy} = (\mu + \eta\partial_t)(\partial_y + \partial_x v_y) \tag{3}$$

Eqs. (1)-(3) were expressed as Eqs. (4)-(6) by using difference method. In the simulation, staggered grid shown in **Fig.1** was defined. Stress was applied with bursted sine wave (500 Hz, 6 waves) to an isotropic and homogeneous medium and generated shear wave was observed. Young's moduli E of 50 and 75 kPa and viscosity η of 0.22 Pa・s were used, where $E = 3\mu$. Vibration with the frequency of 500 Hz was added at t = 15 ms. Vibration source is shown in **Fig.1** and simulation area is shown in **Fig.2**. Particle velocityes at x = 0 and 5 mm were observed and propagation speeds of shear wave were calculated.

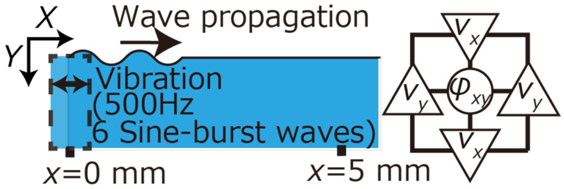

Fig. 1 The subject of the simulation and diagram of staggered grid.

$$\mathrm{v}_{\mathrm{x}}^{n+1}\big|_{i,j,k} = v_x^n\big|_{i,j,k} + \frac{\Delta t}{\rho\Delta y}\left(\sigma_{xy}^n\big|_{i,j+\frac{1}{2},k} - \sigma_{xy}^n\big|_{i,j-\frac{1}{2},k}\right) \tag{4}$$

$$\mathrm{v}_y^{n+1}\big|_{i,j,k} = v_y^n\big|_{i,j,k} + \frac{\Delta t}{\rho\Delta x}\left(\sigma_{xy}^n\big|_{i,j+\frac{1}{2},k} - \sigma_{xy}^n\big|_{i,j-\frac{1}{2},k}\right) \tag{5}$$

$$\begin{aligned}\sigma_{xy}^{n+1}\big|_{i,j+\frac{1}{2},k} &= \sigma_{xy}^n\big|_{i,j+\frac{1}{2},k} \\ &+ \frac{\Delta t}{\mu\Delta y}\left(v_x^{n+1}\big|_{i,j+1,k} - v_x^{n+1}\big|_{i,j,k}\right) \\ &+ \frac{\Delta t}{\mu\Delta x}\left(v_y^{n+1}\big|_{i+\frac{1}{2},j+\frac{1}{2},k} - v_y^{n+1}\big|_{i-\frac{1}{2},j+\frac{1}{2},k}\right) \\ &+ \frac{\eta}{\Delta y}\left(v_x^{n+1}\big|_{i,j+1,k} - v_x^{n+1}\big|_{i,j,k}\right) \\ &- \frac{\eta}{\Delta y}\left(v_x^{n}\big|_{i,j+1,k} - v_x^{n}\big|_{i,j,k}\right) \\ &+ \frac{\eta}{\Delta x}\left(v_y^{n+1}\big|_{i+\frac{1}{2},j+\frac{1}{2},k} - v_x^{n+1}\big|_{i-\frac{1}{2},j+\frac{1}{2},k}\right) \\ &- \frac{\eta}{\Delta x}\left(v_y^{n}\big|_{i+\frac{1}{2},j+\frac{1}{2},k} - v_y^{n}\big|_{i-\frac{1}{2},j+\frac{1}{2},k}\right)\end{aligned} \tag{6}$$

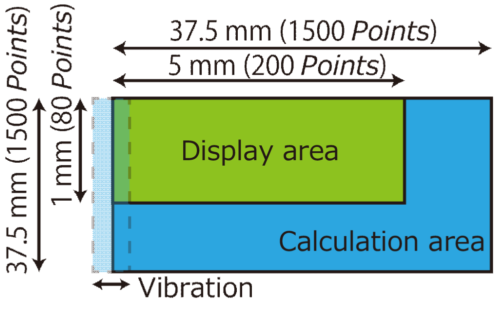

Fig. 2 Area of FDTD simulation.

Courant constant α, which is stable condition, was calculated with Eq. (7).

$$\alpha = \frac{c\Delta t}{\Delta h} \leq \frac{1}{\sqrt{2}} \tag{7}$$

Here, c = shear wave speed, $\Delta h = 2.5 \times 10^{-5}$ m, $\Delta t = 2.5 \times 10^{-6}$ s, and α = 0.5(@75 kPa) were

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used.

3. Experimentation of Young's moduli with OCT

Experiment was conducted by using OCT system in the almost same condition with FDTD simulation. Experimental setup is shown in **Fig. 3**.

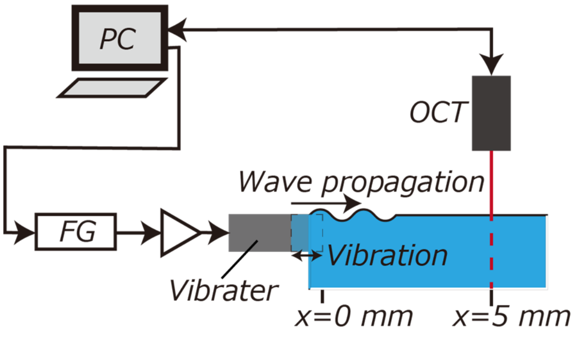

Fig. 3 Experimental system of OCE.

Elastic phantoms of 50 and 75 kPa, size of 10 x 10 x 5 cm^3 (OST) were used as specimens. A matal plate with thicknes of 1 mm (depth direction) and width of 50 mm (deep diection) was connected a vibrator. The plate was set to the specimens so that vibration sourse is a line sourse. Shear waves were measured by OCT system (santec, IVS-2000) and sound sppeds were estimated.

To syncronize start time of vibration and measuring time of the OCT, we modified the OCT system. A data collection borad (NI, PCIe-6320) and BNC connector (NI, BNC-2110) were used to syncronize a function generator, witch is connected an amp and the vibrator, and data acquisition sytem (software: NI, Labview) installed in a PC. The system was modified as following. Timing pulse was applied to from the PC to the function generator. At the same time, data acquisiton was started in the PC. The program was also modified so that one line acquisition (M-mode) at any line x. Using the system, Vibrations were measued at $x = 0$ and 5 mm, where vibration was added at $t = 15$ ms. M-mode data were aquired by the symclonized OCT system. Propagation speeds of shear waves were calculated from arrival time at each point. Young's modulus was calculated by Eq. (8).

$$E = 3\rho c^2, \qquad (8)$$

wher ρ is density and c is propagation speed.

4. Results

Fig. 4 shows FDTD simulation results and **Fig. 5** shows M-mode images acquired by the OCT system. Since **Fig. 4** illustrates $v = v_x^2 + v_y^2$, the wave numbers of FDTD simulation were twice those of OCT images. From the reulsts, we could sucecfully produce shear wave in FDTD simlation and observed waves with the syncronized OCT system. Summarized Young's moduli are shown in **Table 1**. Young's moduli were almost the same in the calculation and the experiment.

5 . Summary

We found that FDTD simulation and the syncronized OCT system worked almost properly to estimate Young's modulus. Vistocity and inhomogeneous media will be considered as future study.

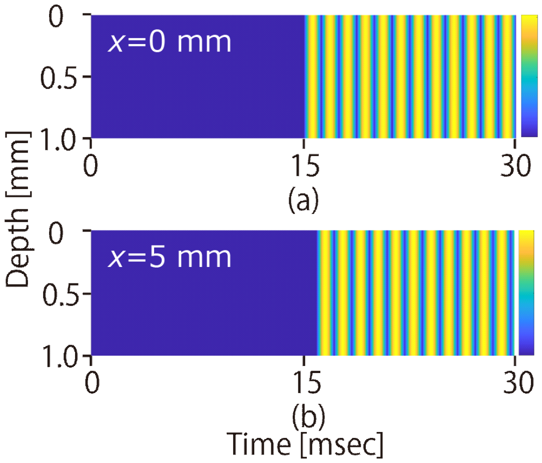

Fig. 4 Change of particle velocity with time.

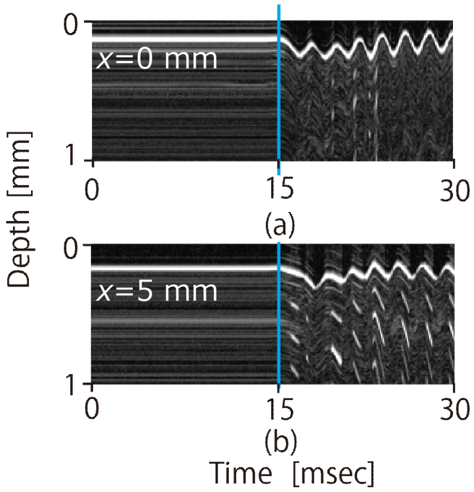

Fig. 5 Change of OCT image with time.

Table 1 Summary of Young's moduli

	FDTD	OCT
75 kPa	75 kPa	75 kPa
50 kPa	51 kPa	52 kPa

References

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