2P2-9

Three-dimensional Numerical Acoustic Simulation with Background Flow Using Method of Characteristics

特性曲線法による媒質の移流を考慮した 3 次元音響伝搬数値解析

Akihiro Fukuda[1†], Kan Okubo[1], Takuya Oshima[2], Takao Tsuchiya[3], Masashi Kanamori[4] ([1]Grad. School System Design, Tokyo Met. Univ.; [2]Facult. Eng., Niigata Univ.; [3]Facult. Sci. Eng., Doshisha Univ.; [4]JAXA.;)

福田 晃大 [1†], 大久保 寛 [1], 大嶋 拓也 [2], 土屋 隆生 [3], 金森 正史 [4] ([1] 首都大院 システムデザイン, [2] 新潟大 工, [3] 同志社大 理工, [4] 宇宙航空研究開発機構)

1. Introduction

Our study group is studying the acoustic simulation with background flow using the constrained interpolation profile(CIP) method, as one of the techniques for predicting the acoustic enviroment around the aircraft and the others.

At present, as a numerical analysis method with background flow, the linearized euler equation (LEE) method incorporating background flow effect in the basic equation of the finite-difference time-domain(FDTD) methid is introduced[1-3]. However, in the LEE method, the faster background flow speed is, the more numerical dispersion occurs, and therefore numerical accuracy decreases as background flow speed increases in some case[4].

The CIP method explicitly computes physical quantities with spatial differential values such as sound pressure. Therefore, it yields a low-dispersive scheme for the simulation of sound field propagation with background flow.[3].

In this study, we examine the analysis method using the type-C CIP method and the type-M CIP method in the three-dimensional acoustic field with background flow. In addition, we compared these method with other methods.

2. Numerical Analysis Using CIP Method

Linearized governing equations of sound field are given in Eq. (1).

$$\rho\frac{\partial \vec{v}}{\partial t} = -\nabla \cdot p,\ \ \nabla \cdot \vec{v} = -\frac{1}{K}\frac{\partial p}{\partial t}\ . \qquad (1)$$

In these equations, ρ denotes the density of the medium, K is the bulk modulus, p is the sound pressure, and $\vec{v} = (v_x, v_y, v_z)$ is the particle velocity.

Here, assuming the analysis of sound field propagation of the x-direction, we can obtain the following equations from Eq. (1).

$$\frac{\partial}{\partial t}p + c\frac{\partial}{\partial x}Zv_x = 0,\ \frac{\partial}{\partial t}Zv_x + c\frac{\partial}{\partial x}p = 0. \qquad (2)$$

In these equations, Z indicates the characteristic impedance (i.e.$Z = \sqrt{K\rho}$) and c represents the sound velocity in the medium (i.e.$c = \sqrt{K/\rho}$).

Then, by addition and subtraction of Eq. (2), the advection equations are given as

$$\frac{\partial}{\partial t}(p \pm Zv_x) \pm c\frac{\partial}{\partial x}(p \pm Zv_x) = 0. \qquad (3)$$

In the CIP method, the spatial differentiation is incorporated into the calculation by partially differentiating Eq. (3) with x, y and z.

When we partially differentiate Eq. (3) with x, y, xy, z, zx, yz and xyz, characteristics of $F_{x\pm}=p\pm Zv_x$, $G_{x\pm}=\partial_x p\pm Z\partial_x v_x$, $H_{x\pm}=\partial_y p\pm Z\partial_y v_x$, $I_{x\pm}=\partial_{xy} p\pm Z\partial_{xy} v_x$, $J_{x\pm}=\partial_z p\pm Z\partial_z v_x$, $K_{x\pm}=\partial_{zx} p\pm Z\partial_{zx} v_x$, $L_{x\pm}=\partial_{yz} p\pm Z\partial_{yz} v_x$, and $L_{x\pm}=\partial_{xyz} p\pm Z\partial_{xyz} v_x$ are obtained in the x direction.

Based on these procedure, the type-C CIP method calculates by using cubic Hermite interpolation. In the type-M CIP method, $H_{x\pm}$ and $J_{x\pm}$ are calculated using linear interpolation. And characteristics of $I_{x\pm}$, $K_{x\pm}$, $L_{x\pm}$ and $M_{x\pm}$ are not used for the type-M method.

3. The CIP method with background flow

Considering background flow on the acoustic field, it is possible to calculate by incorporating background flow speed $\boldsymbol{U} = (U_x, U_y, U_z,)$ into the sound velocity c used for the interpolation calculation of the characteristics.

We calculate $F_{x\pm}$ of next time step using the cubic Hermite interpolation.

$$F_{x\pm}^{n+1}(i) = \begin{matrix} C_{1\pm}F_{x\pm}^{n}(i\mp 1) + C_{2\pm}F_{x\pm}^{n}(i) \\ +C_{3+}G_{x+}^{n}(i\mp 1) + C_{4+}G_{x+}^{n}(i) \end{matrix} \qquad (4)$$

Also, in the type-M CIP method, we calculate $H_{x\pm}$ of next time step using the linear interpolation.

$$H_{x\pm}^{n+1}(i) = C_{1\pm}^{L}H_{x\pm}^{n}(i\mp 1) + C_{2\pm}^{L}H_{x\pm}^{n}(i) \qquad (5)$$

In these equations,

$$\begin{gathered} C_{1\pm} = -2\chi^3 + 3\chi^2,\ \ C_{2\pm} = -2\chi^3 + 3\chi^2 + 1 \\ C_{3\pm} = \xi_{\pm}(\chi^2 - \chi),\ \ C_{4\pm} = \xi_{\pm}(\chi^2 - \chi + 1) \end{gathered} \qquad (6)$$

$$C_{1\pm}^{L} = \chi,\quad C_{2\pm}^{L} = 1 - \chi \qquad (7)$$

$$\xi_{\pm} = \mp(c \pm U_x)\Delta t,\ \ \chi = (c \pm U_x)\Delta t/\Delta x \qquad (8)$$

Calculation of other characteristics are also the same procedure as Eq. (4) and (5).

On the other hand, in the usual CIP method, for example in calculation in the x direction, the particle velocity in the perpendicular direction v_y and v_z are not included in the calculation. However, when considering background flow, it is necessary to calculate the propagation of the advection direction also in these particle velocity.

In this calculation, it can be obtained using cubic Hermite interpolation and linear interpolation like Eq. (4) and (5). We calculate v_y of next time step using the cubic Hermite interpolation.

$$v_y^{n+1}(i) = \begin{array}{c} C_{1\pm} v_y^n(i \mp 1) + C_{2\pm} v_y^n(i) \\ + C_{3\pm} \partial_x v_y^n(i \mp 1) + C_{4\pm} \partial_x v_y^n(i). \end{array} \tag{9}$$

Additionally, in the type-M CIP method we calculate $H_{x\pm}$ of next time step using the linear interpolation.

$$\partial_y v_y^{n+1}(i) = C_{1\pm}^L \partial_y v_y^n(i \mp 1) + C_{2\pm}^L \partial_y v_y^n(i) \tag{10}$$

Here, the coefficient $C_\pm$ and $C^L{}_\pm$ is the same as Eq. (6), (7) and (8). However, ξ and χ are expressed as follows.

$$\xi_\pm = \mp U_x \Delta t, \quad \chi = U_x \Delta t / \Delta x \tag{11}$$

The sign of ξ depends on background flow direction.

Calculation of differential values of the particle velocity in the perpendicular direction are also the same procedure as Eq. (9) and (10).

4. Computational results

For the parameters in this analysis, the grid size is $\Delta x = \Delta y = \Delta z = 0.06$m, time step is $\Delta t = 60\mu$s, the number of grid is $NX = NY = NZ = 201$, the number of time step is $NT = 200$ and sound speed is $c = 343$m/s.

Figures 1 and **2** shows three-dimensional acoustic propagation simulation with background flow speed $(U_x, U_y, U_z,) = (-0.3c,\ 0.1c,\ 0.3c)$ using the type-C CIP method (Fig.1) and the type-M CIP method (Fig.2) by sound pressure distribution. From Figs. 1 and 2, we ascertain the sound wave propagates considering the background flow.

Figure 3 shows time-pressure waveform at the sound receiving point (7.38m, 7.38m, 7.38m) using the type-C CIP method, the type-M CIP method, the LEE method and exact solution[5] with background flow speed $U_x = U_y = U_z = 0.3c$. Here, finer grids (0.02m and 0.03m) are also used in calculation by the LEE method. From Fig. 3, the LEE method shows a large error from exact solution. However, the type-C CIP method and type-M CIP method is almost consistent with the exact solution.

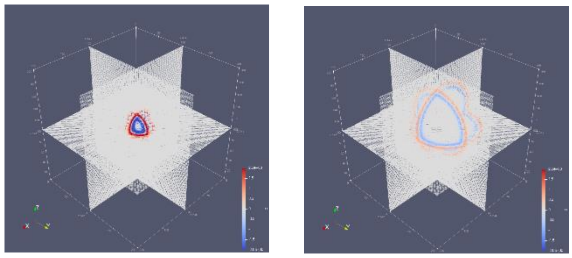
Finally, **Figure 4** shows the comparison of calculation time by each method. The calculation environment uses OpenMP, and the number of threads is set to 8. From Fig.4, the type-M CIP method had the calculation time of about 0.62 times than that of the type-C method.

5. Conclusion

We examined the three-dimensional acoustic simulation with background flow using the CIP method. These results suggest that CIP analysis provides higher accuracy for calculating the propagation effect with background flow than that obtained using the conventional scheme.

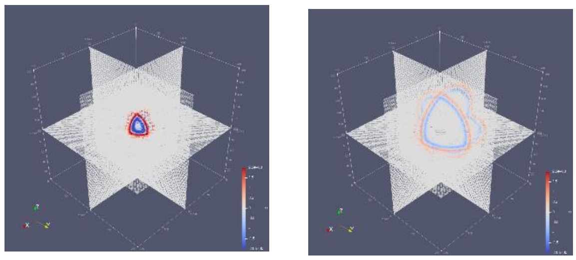
References

1. T. Tsuchiya, K. Okubo, and N. Takeuchi: *J. Acoust. Soc. Jpn* 64 (2008) 443.
2. T. Sakuma, S. Sakamoto and T. Otsuru: ‘Computational Simulation in Architectural and Environmental Acoustics’. Springer, 2014.
3. T. Oshima, M. Imano, Y. Hiraguri and Y. Kamoshida: *Applied Acoustic74* (2013). *1354-1366*.
4. Fukuda, et. al, USE 2016 3P2-6.
5. C. Bogey and C. Bailly: Acta Acustica united with Acustica(2002), *463-471*.



(a) t = 50Δt [s] (b) t = 150Δt [s]

Fig. 1 Spatial distribution of sound pressure used in the type-C CIP method.



(a) t = 50Δt [s] (b) t = 150Δt [s]

Fig. 2 Spatial distribution of sound pressure used in the type-M CIP method.

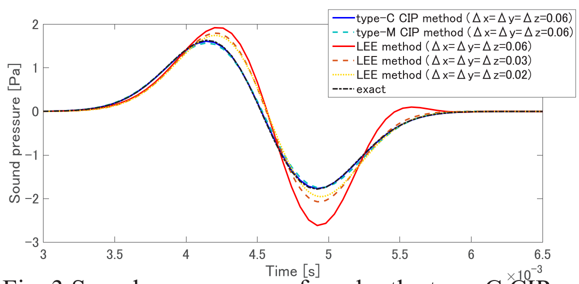


Fig. 3 Sound pressure waveform by the type-C CIP method, the type-M CIP method, the LEE method and exact solution.

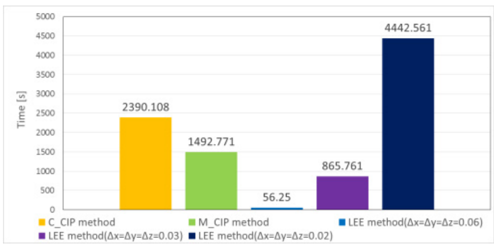


Fig. 4 Comparison of calculation time.