3P5-10

Investigation on maximum likelihood method for measurement of regional pulse wave velocity
脈波の局所伝搬速度計測を目指した最尤推定法に関する検討

Hideyuki Hasegawa[†]
(Graduate School of Science and Engineering for Research, University of Toyama)
長谷川英之[†] （富山大院 理工）

1. Introduction

It is important to evaluate atherosclerosis in the early stage for prevention of serious cardiovascular events, such as myocardial infarction and stroke. The elasticity of the arterial wall is a useful marker for diagnosis of atherosclerosis because it is altered by atherosclerosis [1]. For evaluation of the elasticity of the arterial wall, the pulse wave velocity (PWV) method is developed as a non-invasive method for evaluation of the elasticity of the arterial wall [2-4]. In the conventional PWV method, the PWV is measured between distant two points, such as two points in brachial and ankle arteries, and the average elasticity between such distant two points is evaluated. However, the elasticity of the arterial wall is different region by region and atherosclerotic lesion is formed locally. Therefore, the measurement of the regional PWV is preferred. However, the pulse wave propagates along the artery relatively fast (several m/s), and the measurement of the regional PWV is requires high temporal resolution because the time delays between pulse waves measured at two points becomes less than 10 ms when the PWV is measured in a short segment of the artery of several centimeters in length.

High frame rate ultrasound, which enables an imaging frame rate of over one thousand Hz, is preferable for measurement of the regional PWV [5,6]. In this report, the regional PWV was estimated using the acceleration waveforms measured from the arterial wall by high frame rate ultrasound. Also, methods for estimation of the temporal frequency and wavenumber of the measured acceleration field were developed.

2. Materials and Methods

2.1 Estimation of temporal frequency

In the present study, the acceleration waveforms on the arterial wall were measured at a very high frame rate of 1302 frames per second (fps), where m denotes the scan line number. The position of point of interest in each scan line is defined as $\mathbf{r}_m = (x_m, z_m)$, where x_m and z_m are the lateral and axial positions.

In the present study, the Hilbert transform was applied to the measured acceleration waveforms to obtain the analytic signals $a_m(t)$ at the m-th point of interest. The mean temporal frequency of the acceleration waveform f_0 at t is estimated as follows [7]:

$$f_0(t) = \frac{1}{T_s} \frac{\angle \gamma(t)}{2\pi}, \tag{1}$$

$$\gamma(t) = \sum_{m=0}^{M-1} a_m(t + T_s) \cdot a_m^*(t), \tag{2}$$

where M and * denote the number of scan lines and complex conjugate, respectively.

2.2 Wavenumber estimation

The complex acceleration measured at each point of interest $\mathbf{r}_m$ and time t is composed in a vector from:

$$\mathbf{a} = [a_0(t)\ a_1(t)\ \cdots\ a_{M-1}]^{\mathrm{T}}, \tag{3}$$

where $^{\mathrm{T}}$ denotes transpose.

By assuming that the pulse wave propagates at a speed of c_{PWV} in the measured segment, the measured acceleration $\mathbf{a}$ can be modelled as follows:

$$\hat{\mathbf{a}} = A(k, t) \cdot \mathbf{h} + \mathbf{n}, \tag{4}$$

where k and $\mathbf{n}$ are the wavenumber of the pulse wave and noise vector, respectively, and $A(k, t)$ is the acceleration at the source (corresponding to the acceleration at the 0-th point of interest $\mathbf{r}_0$). The vector $\mathbf{h}$ denotes the phase shift of the complex acceleration corresponding to the time delay owing to the pulse wave propagation along the assigned path $[\mathbf{r}_0\ \mathbf{r}_1\ \cdots\ \mathbf{r}_{M-1}]$. The vector $\mathbf{h}$ is expressed as follows:

$$\mathbf{h} = \left[e^{jkd_0}\ e^{jkd_1}\ \cdots\ e^{jkd_{M-1}}\right]^{\mathrm{T}}, \tag{5}$$

where k is the wavenumber of the pulse wave, and d_m is the distance of the m-th point of interest $\mathbf{r}_m$ from $\mathbf{r}_0$ along the assigned path $[\mathbf{r}_0\ \mathbf{r}_1\ \cdots\ \mathbf{r}_{M-1}]$ expressed as follows:

$$d_m = \sum_{i=0}^{m} |r_i - r_{i-1}|, \tag{6}$$

By assuming that noise $\mathbf{n}$ is Gaussian, the likelihood is defined as follows:

----

†hasegawa@eng.u-toyama.ac.jp

$$p(\mathbf{a}|\breve{a}_0) = \frac{1}{\det(\pi\mathbf{K})} \times \exp\{-(\mathbf{a} - A\mathbf{h})^{\mathrm{H}}\mathbf{K}^{-1}(\mathbf{a} - A\mathbf{h})\}, \quad (7)$$

where $^{\mathrm{H}}$ denote Hermite transpose, and $\mathbf{K} = \mathbf{a}\mathbf{a}^{\mathrm{H}}$.

The estimate $A(k,t)$ of the acceleration at the source position $\mathbf{r}_0$, which maximizes the likelihood, is obtained as follows:

$$A(k,t) = \frac{\mathbf{K}^{-1}\mathbf{h}}{\mathbf{h}^{\mathrm{H}}\mathbf{K}^{-1}\mathbf{h}} \cdot \mathbf{a}. \quad (8)$$

3. Simulation Experimental Results

In the present study, the proposed method was evaluated by the simulation experiments. The simulated acceleration waveform is shown in **Fig. 1**(a). The pulse wave assumed to propagate along a straight artery at a speed of 2.5 m/s. In the simulated acceleration field shown in Fig. 1(b), the change in the arrival time of the pulse wave can be seen. Figure 1(c) shows the phases of the analytic signals of the simulated acceleration waveforms.

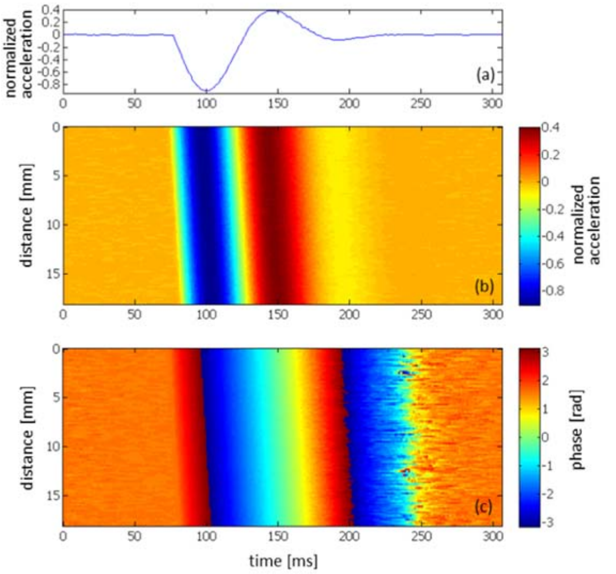

Fig. 1: Simulated acceleration waveform. (a) Waveform at the central position. (b) Acceleration shown as a function of distance and time. (c) Phases of analytic signals of acceleration waveforms.

The proposed method was applied to the analytic signals of the simulated acceleration waveforms. **Figure 2**(a) shows the estimated temporal frequency of the simulated acceleration waveform. Stable estimates are obtained during the time period when the pulse wave present. Figure 2(b) shows $A(k, t)$ obtained by the proposed method on the basis of maximum likelihood estimation. From the peak position in Fig. 2(b), the wavenumber (corresponding to the spatial frequency) of the pulse wave can be determined.

Figure 3 shows a profile of $A(k, t)$ plotted as a function of the spatial frequency obtained at a time when $A(k, t)$ is maximum. From Fig. 3, the spatial frequency of the pulse wave is determined as -3.65 m^{-1}. The temporal frequency of the pulse wave at the same point of time is determined as 9.27 Hz from Fig. 2(a). The PWV is estimated by dividing the temporal frequency by the spatial frequency. The PWV was determined as 2.54 m/s, which is in good agreement with the true PWV of 2.5 m/s.

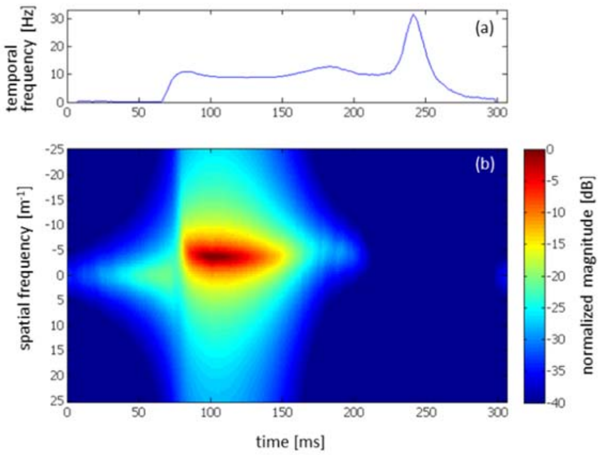

Fig. 2: Simulation experimental results. (a) Mean temporal frequency of acceleration waveforms. (b) Estimated magnitude of source of acceleration $A(k, t)$.

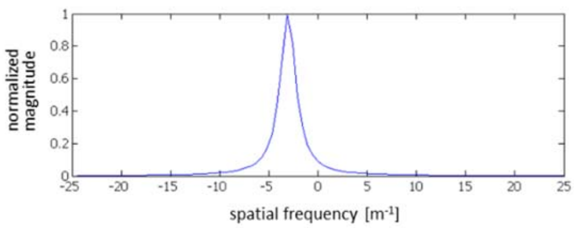

Fig. 3: Profile of $A(k, t)$ at peak.

6. Conclusion

In the present study, a method on the basis of maximum likelihood estimation was proposed for estimation of the regional PWV.

References

1. I. Wendelhag, O. Wiklund, and J. Wikstrand: Arterioscler. Thromb. Vasc. Biol. 16 (1996) 843.
2. P. Hallock: Arch. Int. Med. 54 (1934) 770.
3. T. Imura, K. Yamamoto, K. Kanamori, et al.: Cardiovasc. Res. 20 (1986) 208.
4. B. Li, H. Gao, Y. Liu, et al.: Hypertens. Res. 29 (2006) 309.
5. H. Hasegawa, K. Hongo, and H. Kanai: J. Med. Ultrason. 40 (2013) 91.
6. H. Hasegawa, M. Sato, and T. Irie: Jpn. J. Appl. Phys. 55 (2016) 07KF01.
7. H. Hasegawa and H. Kanai: IEEE Trans. Ultrason. Ferroelect. Freq. Contr. 56 (2009) 2450.