

New Phase Matching Method for Ultrasonic Tissue Displacement Measurement

超音波組織変位計測のための新しい位相マッチング法

Chikayoshi Sumi (Dept. of Info. & Commun. Sci., Sophia Univ.)

炭 親良 (上智大学 情報理工学科)

1. Introduction

We have been developing high accuracy ultrasonic tissue displacement vector measurement methods, for instance, multidimensional cross-spectrum phase gradient method (MCSPGM),^{1,2)} multidimensional autocorrelation method (MAM),^{3,4)} multidimensional Doppler method (MDM),^{3,4)} digital demodulation method,⁵⁾ lateral modulation method,^{3,4)} spectral frequency division method.⁶⁾ The simultaneous equations are solved, or the over-determined systems derived from over-determined plural waves physically generated or artificially generated by the spectral frequency division method are also solved for increasing the measurement accuracy.⁷⁻⁹⁾ These methods allow the measurement of a tissue strain tensor as well as medium displacements of shear wave, i.e., low frequency deformations generated by static compression/stretching, applying vibrations or radiation forces, etc.

For the measurements, the phase matching method using the multidimensional cross-correlation method is also important.¹⁻³⁾ This method opened up the ultrasonic tissue elasticity measurement field in that the tissue strain measurement was enabled. The method is required even when the tissue displacement is infinitesimal, and the method also allows omitting phase unwrapping processing when the tissue displacement is large. However, the original phase matching method leads to a discontinuous estimate of displacement distribution when performing spatially stationary statistical processings on raw phase data during the tissue displacement measurement or when implementing various optimizations onto the systems, e.g., the regularizations, the weighting least squared estimations using the statistics. This report describes a new phase matching method to cope with the error generations.

2. Method

2.1 Generations of errors

The cross-correlation method yields a coarse

local estimate of displacement from a paired local rf-echo signals, i.e., an estimate digitized by the sampling intervals of echo signals.^{1,2)} After spatially shifting local echo data with the coarse estimate such that the corresponding local rf-echo signals overlap more, phases such as a cross-spectrum phase, a phase of complex auto-correlation function and a phase difference in analytic signal phases are respectively calculated for the paired rf-echo signals in MCSPGM, MAM and MDM. In MAM, the phase corresponds to the difference in the instantaneous phases of paired rf-echo signals, and in order to stabilize the estimate, the moving-averaging is performed on the calculated phase directly (MAM-i) or on the complex auto-correlation function before calculating the phase (MAM-ii). Both the calculated phases have discontinuous distributions and then in the former calculation, the moving-averaging yields errors. In MDM, the errors occur similarly. In MCSPGM and the latter calculation in MAM yields no errors.

Moreover, various optimizations yield errors on all the displacement measurement methods. For instance, the maximum likelihood (ML) estimate using covariance matrices of the erroneous phases also leads to errors [ML estimate can be performed with MAP (*maximum a posteriori*)]. The regularization¹⁰⁾ or weighting least-squared estimations¹¹⁾ using the variances of displacement measurements also leads to errors (*a posteriori* processing) if the displacement measurements are erroneous due to the erroneous phases. Moreover, the statistical estimates are to be used on the equations for the unknown residual displacements to be added to the coarse estimates. Alternatively, using the variance estimated in the original 1-dimensional autocorrelation method¹²⁾ and used for the power Doppler, the variance similarly estimated using multidimensional moving-averaging for MAM¹³⁾ or the Ziv-Zakai Lower Bound¹¹⁾ also leads to the same error, although the variances are not estimated under the assumption of a stationary process (i.e., *a priori* processing).

Thus, all the errors are caused by performing at least one processing with respect to the residual displacements.

2.2 New phase matching method

To cope with the generations of the errors, the phase matching is directly performed on the calculated phases. For MCSPGM, the equation for the unknown residual displacements (U_x, U_y, U_z) is expressed as

$$F_x U_x + F_y U_y + F_z U_z = \theta, \quad (1)$$

where (F_x, F_y, F_z) and θ respectively express frequencies within the bandwidth of rf-echo signals and the cross-spectrum phase of local rf-echo signals; and for both MAMs or MDM, the equation for (U_x, U_y, U_z) is expressed as eq. (1) by considering (F_x, F_y, F_z) and θ as moving-averaged instantaneous frequencies and the instantaneous phase difference, respectively. In each case, by adding the phase corresponding to the coarse estimates (D_x0, D_y0, D_z0) to θ by

$$\theta' = \theta + F_x D_x0 + F_y D_y0 + F_z D_z0, \quad (2)$$

the equation directly for the unknown target displacements (D_x, D_y, D_z) corresponding to (D_x0, D_y0, D_z0) + (U_x, U_y, U_z) can be obtained using the spatially continuous θ' as follows:

$$F_x D_x + F_y D_y + F_z D_z = \theta'. \quad (3)$$

2.3 Maximum likelihood (ML) estimation

In this report, the new phase matching is applied to the ML estimation. The over-determined systems are generated at each point of interest in a region of interest by using plural waves with different steering angles or by performing the spectral frequency division. When the over-determined system is expressed as

$$\mathbf{F}\mathbf{D} = \boldsymbol{\theta}, \quad (4)$$

where \mathbf{F} expresses a matrix of frequencies; and \mathbf{D} and $\boldsymbol{\theta}$ respectively express the vectors of unknown target displacements and the phases, the ML estimates are obtained by solving

$$\mathbf{F}^T \mathbf{C}^{-1} \mathbf{F} \mathbf{D} = \mathbf{F}^T \mathbf{C}^{-1} \boldsymbol{\theta}, \quad (5)$$

where \mathbf{C} is the covariance matrix of the matched phases and T denotes a transpose of matrix.

3. Phantom experiments (a lateral compression)

The synthetic aperture echo data used in ref. [4, 6-9] obtained from the agar phantom [40 (axial) \times 96 (lateral) \times 40 (elevational) mm] having a central circular cylindrical inclusion (diameter, 10 mm; depth, 19 mm) with a shear modulus different from that of the surrounding region. Elasticity was controlled by adjusting the concentration of the agar powder (Wako Pure Chemical Industries, Ltd., 010-15815, Osaka, Japan; 6.0 and 3.0%). To

control US attenuation, graphite powder (Wako Pure Chemical Industries, Ltd., 070-01325, concentration, 3.0%) was also added. The resultant phantom had shear moduli of 2.63 and 0.80×10^6 N/m² in the inclusion and surrounding regions, respectively (the relative shear modulus, 3.29). Manually, the phantom was compressed by 2.0 mm in the lateral direction.

An over-determined system was obtained using MAM-ii from generated four beams with steering angles $\pm 30^\circ$ and $\pm 40^\circ$ and with a parabolic apodization (Case 1). In addition, a frontal beam (0°) was also generated with a rectangular apodization, which was used for yielding another system (MAM-ii) from four quasi-beams generated by dividing the corresponding original spectra in a vertical or horizontal direction symmetrically (Case 2). The SNRs of measurement results were estimated at the central position of inclusion by calculating the ratios of means and SDs. In Case 1, the SNRs of magnitude and angle of displacement vector respectively increased from simple least squared results, 82.0 and 161, upto 83.0 and 167; and in Case 2 the SNR of lateral strain from 0.199 upto 0.214 (both SNRs of axial strains, 0.106).

4. Conclusions

The new phase matching method yielded system of equations directly for unknown displacements, which allowed performing the accuracy LM estimations with no errors. Other optimization results will also be reported elsewhere.

References

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