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Numerical Nonlinear Formulation of Rott Equations for a Thermoacoustic Engine: Stokes Drift and Phase Change

熱音響エンジンにおけるロット方程式の数値的非線形化：ストークス速度と相変化

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1. Introduction

Sound wave is generated in narrow tubes of a stack in a thermoacoustic engine when there is sufficient temperature gradient along narrow tubes [1]. Acoustic streaming occurs inisde narrow tubes associated with sound wave propagation in narrow tubes. However, little is known on acoustic streaming inside narrow tubes although there are many studies on acoustic streaming outside of a stack [1, 2]. Study on acoustic streaming is important because acoustic streaming generally reduces efficiency of a thermoacoustic engine by unwanted heat transfer. In the present paper, acoustic streaming in a narrow tube is studied by numerical nonlinear formulation of the linear Rott equations for a thermoacoustic engine. Rott equations are known as a standard theory of thermoacoustics [1, 3].

Tsuda and Ueda [4] experimentally reported that the critical temperature difference between two ends of a stack for sound generation was decreased from about 150 °C for a dry stack to about 50 °C for a stack wetted with water vapor. The authors [5] have already reported results of numerical simulations on evaporation and condensation of water vapor in a wet stack. In the theoretical model, the linear Rott equations are used. Although the effect of temperature gradient was taken into account in the numerical simulations of evaporation and condensation as well as thermal conduction between a fluid (gas+vapor) parcel and wall of a stack, the temperature gradient was set as zero in Rott equations by a mistake in Ref. [5]. Thus in the present paper, refined numerical simulations are also performed.

2. Model

The present model is based on a Lagrangian point of view [5]. In other words, translational motion of a fluid parcel is numerically simulated. Furthermore, expansion and contraction of a fluid parcel is numerically simulated associated with acoustic pressure oscillation. The effect of non-equilibrium evaporation and condensation in a wet stack is also taken into account. Rott equations are used in order to calculate instantaneous acoustic pressure and particle velocity in a fluid parcel. In the Rott equations, stationary state is assumed. Thus amplitudes of acoustic pressure and particle velocity are given solely as a function of position in a narrow tube of a stack based on an Eulerian point of view. Thus the use of Rott equations in a model based on a Lagrangian point of view is justified. The equations of the present model are as follows.

$$x(t+\Delta t) = x(t) + u\cdot\Delta t \qquad (1)$$

where $x(t)$ is position of a fluid parcel along a straight tube of a stack at time t, u is instantaneous translational velocity of a fluid parcel, and Δt is a small time step in the numerical simulations. The instantaneous translational velocity of a fluid parcel at time $t+\Delta t$ is calculated as follows.

$$u(t) = u(x(t)) \qquad (2)$$

$$u(t+\Delta t) = \mathrm{Re}\left(u_1\big(x(t+\Delta t)\big)e^{i\omega(t+\Delta t)}\right) \qquad (3)$$

$$u_1\big(x(t+\Delta t)\big) = u_1\big(x(t)\big) + \frac{\partial u_1}{\partial x}\Delta x \qquad (4)$$

where $u_1\big(x(t)\big)$ is the amplitude of particle velocity expressed with a complex number at position $x(t)$, ω is angular frequency of an acoustic wave, and $\Delta x = u\cdot\Delta t$ is a displacement of a fluid parcel in time Δt. The second term in the right hand side of Eq. (4) is the nonlinear advective derivative of particle velocity. In general, in fluid dynamics, nonlinear term is originated in the nonlinear advective derivative of particle velocity [6].

3. Results and Discussions

As in the experiment by Tsuda and Ueda [4],

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the acoustic frequency is assumed as 93.8 Hz in the present numerical simulations. Temperature at lower and higher temperature side of a stack is assumed as 25 °C and 225 °C, respectively. The radius of a narrow tube of a stack is assumed as 0.47 mm. In the present study, standing-wave thermoacoustic engine is simulated.

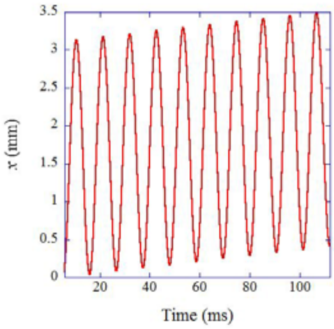
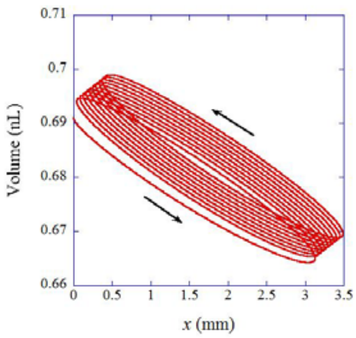


Fig. 1 Position (x) of a fluid parcel as a function of time for ten acoustic cycles.



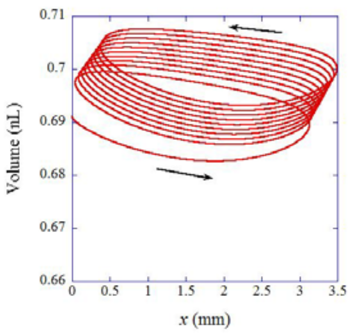


Fig. 2 x-V diagram in dry (upper) and wet (lower) stacks.

A fluid parcel moves forward and backward repeatedly as shown in Fig. 1. Mean position of a fluid parcel gradually shifts to higher temperature side (positive x direction). This is a result of the nonlinear term in Eq. (4). However, nonlinear effect is only partly taken into account in the present model because the mean Eulerian velocity is assumed as zero in Rott equations. Thus the drift in Fig. 1 is solely the Stokes drift [7]. In order to obtain the actual Lagrangian mean velocity which is the mass transport velocity of acoustic streaming, the mean Eulerian velocity should be added to the Stokes drift velocity [7-9].

Time averaged pV work done by a fluid parcel is larger in a wet stack than that in a dry stack, which corresponds to higher acoustic energy. Mean volume of a fluid parcel increases with time because mean position of a fluid parcel moves to higher temperature side of a stack (Fig. 2). In a wet stack, mean volume of a fluid parcel increases more by evaporation. This is the reason for the larger pV work in a wet stack. On the other hand, volume oscillation amplitude decreases by evaporation and condensation in a standing-wave engine (Fig. 2).

4. Conclusion

Stokes drift is numerically simulated by the numerical nonlinear formulation of Rott equations. In order to obtain the mass transport velocity of acoustic streaming, the mean Eulerian velocity should be added to the Stokes drift velocity. Time averaged pV work done by a fluid parcel in a wet stack is larger than that in a dry stack in a standing-wave engine due to more increase in mean volume of a fluid parcel by evaporation.

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