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Phonic crystal based on a Shive wave machine

シャイブ式ウェーブマシンに基づくフォノニック結晶の作製
Masahiro Ino†, Motonobu Tomoda, and Oliver B. Wright
(Faculty of Engineering, Hokkaido University)
猪野真大†，友田基信， Wright Oliver B. (北大　工)

1. Introduction

Shive wave machines[1] are widely used for demonstrating wave phenomena in physics and engineering. They consist of a periodic one-dimensional array of torsionally-coupled rods, along which passes a torsional wave. The Shive wave machines realizes a conveniently low wave speed, in the sub-m/s range, with easily visible amplitude. Various wave phenomena such as reflection, transmission, and standing waves etc. can be viewed.

Shive wave machines are characterized by the moment of inertia of each rod I and the torsional spring constant κ.[2] The equation of motion of the s-th unit in a conventional one-dimensional mass-spring model is governed by (see **Fig. 1**)

$$m\ddot{x}_s = k(x_{s+1} + x_{s-1} - 2x_s), \qquad (1)$$

where m is the mass and k is the spring constant, whereas the equation of motion of the s-th unit in a Shive wave machine is governed by

$$I\ddot{y}_s = \frac{2Td^2}{a}(y_{s+1} + y_{s-1} - 2y_s), \qquad (2)$$

where T is tension of the two wires that provide the torsional coupling, d is the distance from center of the rods to the supporting wires, and a is the lattice constant. One can appreciate the exact correspondence with the mass-spring model.

By the use of a unit cell with two different moments of inertia, it is possible to mimick the behaviour of a phononic crystal,[3] something which has not previously been demonstrated with a Shive wave machine.

In this paper, we demonstrate the physics of a phononic crystal based on a Shive wave machine. In particular we observe phenomenon of vibrational band gaps.

2. Wave-machine design

Fig. 2 shows a photograph of the wave machine. The acrylic rods of square cross section of side 6 mm and length 320 mm are mounted on an axial central wire, and the two wires that provide the torsional coupling are mounted at $d = 6$ mm away from this central axis are also arranged to pass through the rods. Pipe-shaped spacers of length 10 mm are used, providing a unit cell of length $a = 16$ mm. Approximately 100 unit cells are used. The wave machine is mounted on a light and rigid aluminum frame. **Fig. 3** gives a close-up view. All wires are adjusted with guitar pegs at the end of the setup, allowing the tension to be adjusted. We arrange for the speed of travelling wave to be in the range 10 - 40 cm/s, and the wave amplitude to be ~4 cm.

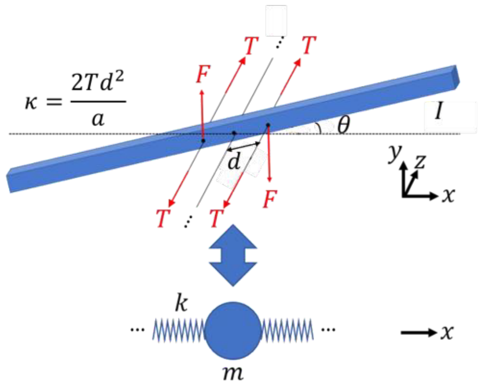

Fig. 1 The correspondence between a Shive wave machine and a mass spring model. Moment of inertia I and torsional constant κ in the torsional case respectively correspond to mass m and spring constant k in the mass-spring model.

Fig. 2 View of the bare wave machine. About 100 unit cells are constructed with acrylic rods and threaded wires. A travelling wave is clearly visible.

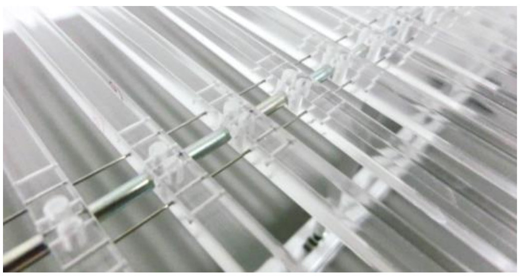

Fig. 3 Close-up view of the wave machine. Pipe-shaped spacers determine the unit cell length. Wires in the side holes generate the restoring forces, allowing the torsional wave to propagate.

A phononic crystal is constructed by periodically varying the moment of inertia of the rods, as shown in **Fig. 4**, by adding extra masses (7.9 g) located with their centres of mass at 5 mm from the ends of the rods. The moment of inertia of the rods without the masses is $I_1 = 1.05 \times 10^{-4}$ kg m^2, whereas with the masses it is $I_2 = 3.13 \times 10^{-4}$ kg m^2, as measured by pendulum experiments.

3. Results and development of tracking system

To monitor the response of the torsional phononic crystal on the excitation frequency, we recorded constant-frequency waves generated with a motor-and-crank. The theoretical dispersion relation can be expressed in the form

$$\boldsymbol{\omega^2 = \frac{4Td^2(I_1+I_2)}{I_1 I_2 a}\left\{1 \pm \sqrt{1 - \frac{4I_1 I_2 \sin^2\left(\frac{qa}{2}\right)}{(I_1+I_2)^2}}\right\}}, \quad (3)$$

where q is the wave number, and the wave velocity in the limit of low frequencies is given by $v_p = \sqrt{2Tad^2/(I_1+I_2)}$. The phase velocity measured from movies such as **Fig. 4(a)** was found to be 0.31 m/s, corresponding to a calculated tension of $T = 4.6$ N. At a frequency of 3.5 Hz, we could verify that adjacent rods oscillated with opposite phase, as shown as **Fig. 4(b)**, clearly indicating Bragg scattering and indicative of a band edge. The predicted frequency of this band edge is $f = \sqrt{(2Td^2/I_2 a\pi^2)}$, which leads to the value 3.8 Hz, in reasonable agreement with experiment. On further increase in frequency to 4.2 Hz, as shown in **Fig. 4(c)**, the wave became heavily damped as expected from the calculated dispersion relation.

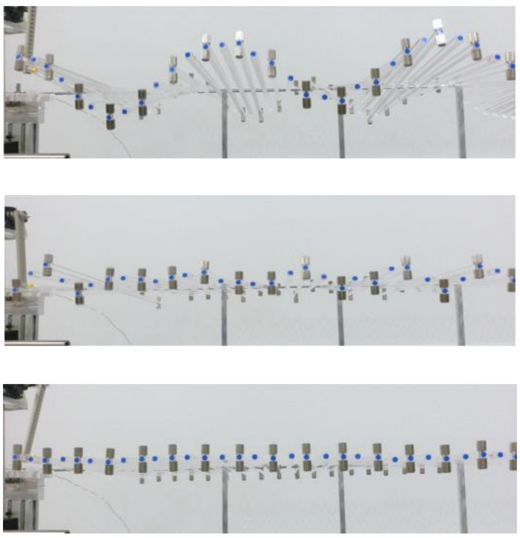

Fig. 4 Frames of constant frequency movies in the phononic crystal (a) 1.5 Hz, (b) at the edge of the Brillouin zone at 3.5 Hz, and (c) in the band gap at 4.2 Hz. To form the phononic crystal, the moment of inertia of the acrylic rods is modulated periodically in space by the addition of masses. The wave is generated with a motor and crank, visible in the upper left.

For systematic measurements, we measured the vibrational displacement of each rod by means of a visual tracking system that could capture the displacement of the rod ends, as shown in **Fig. 5** for the case of the rods without the added masses. This tracking system will be also applied in future to the phononic-crystal case, allowing the dispersion relation and the presence of phononic band gaps to be further quantified.

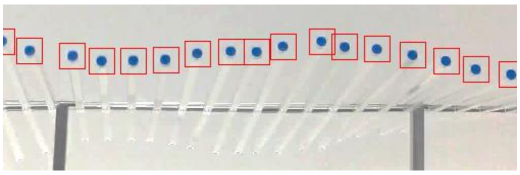

Fig. 5 The red rectangles show automatic identification by the image tracking system. The blue dots at the end of the rods are adhesive colour seals for ease of tracking.

4. Conclusion

In conclusion, we have demonstrated that it is possible to model the behaviour of a torsional phononic crystal with a Shive wave machine. This gives a dramatic visual demonstration of the physics of vibrational effects in periodic media, ideal for educational purposes. In future, it should also be possible to demonstrate the physics of more complicated structures and thereby better understand their dynamics.

References

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