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A mathematical model of the Lamb wave reflection at a two dimensional rectangular notch

2 次元矩形状欠陥における Lamb 波の反射数学モデル

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1. Introduction

Guided wave technique for the nondestructive testing is one of the useful and efficient methods. Due to its long-range propagation behavior, axial locations of defects can be detected effectively by the technique. However, the evaluations of the defects are not yet quantitative because of the complex relations between the defect morphologies and their signals detected. To reveal the complex relations, it is an important step to understand propagation phenomena at a simple rectangular notch. In the previous researches, a simple concept of the reflection phenomena at the rectangular notch has been presented regarding the Lamb wave [1, 2], the SH plate wave [3] and the guided wave in piping [4]. That is, the reflection coefficient at the rectangular notch varies basically due to the interference between the signals generated at the initial and final ends of the notch, which shows the periodic structure with notch length. The reflection phenomena including the concept were investigated well in terms of the FEM and the experimental approaches [1, 2] and the FEM and the theoretical approaches [3].

In this paper, a simple mathematical model at a two dimensional rectangular notch describing the reflection coefficients based on the above concept were formulated regarding the A_0 mode Lamb wave. The experiments were also carried out to evaluate the reflection phenomena at the rectangular notches as functions of their length and depth. The experimental results agreed fairly well with the mathematical model. It was confirmed that the mode conversions at the initial and final ends of the notch took an important role to determine the reflection coefficient.

2. Mathematical model

The simple mathematical model regarding the Lamb wave in the lowest frequency region is presented. Propagation paths at a rectangular notch are shown in **Fig. 1**. We suppose here in the model a preferential transduction of the A_0 mode using the critical angle method. Therefore, the S_0 mode paths were omitted both at the transmitting and receiving points as shown in Fig. 1. As results of the transmissions and reflections accompanying with the mutual mode conversions both at the initial and final ends of the notch, five propagation paths can be observed (multiple reflections between the two ends were omitted because their coefficients were assumed to be much smaller). The wave observed actually is obtained by the superposition of all the five wave-packets depicted in Fig. 1 because a normal defect length is at most smaller than few wavelengths of the Lamb wave. The mathematical formulation for the reflection coefficient at the notch regarding the Lamb wave in the lowest frequency region is

$$R = |r_{AA} + r_{AAAA}\exp(2ik_A L) + (r_{ASAA} + r_{AASA})\exp[i(k_A + k_S)L] + r_{ASSA}\exp(2ik_S L)|, \quad (1)$$

where, k_A, k_S, and L are the wavenumbers of the A_0 and S_0 modes and the length of the rectangular notch, respectively. r_{AA}, r_{AAAA}, r_{ASAA}, r_{AASA} and r_{ASSA} are individual reflection coefficients for the five paths, which are as a function of notch depth. It is difficult to calculate the individual reflection coefficients; measured by the experiments. To measure the individual reflection coefficients, the defect length L has been taken long enough (L=400 mm) to separate the five wave packets. **Figure 2** shows the individual reflection coefficients as a function of notch depth in a 2-mm-thick aluminum alloy plate.

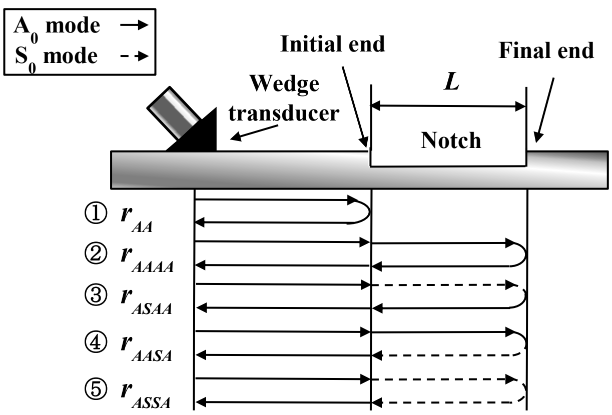

Fig. 1 Propagation path of the Lamb wave at the rectangular notch.

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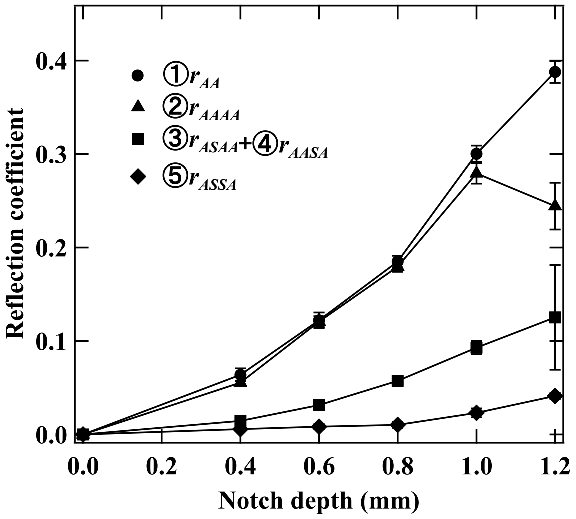

Fig. 2 Individual reflection coefficient.

3. Experimental verification

Experimental setup for the verifications is shown in **Fig. 3.** The wedge transducer for the preferential transduction of the A_0 mode Lamb wave was employed on the 2-mm-thick aluminum plate specimen at three different frequencies: 450, 500, and 550 kHz. The 31-cycle tone burst signal was used in the experiments. Two types of gradually incremental notches were introduced in the plate specimens. One is the notch length incremental (LI) test and the other is the simultaneous incremental (SI) test of both notch length and depth.

In the LI test, the notch depth was constantly at 1 mm (50% of the plate thickness) and the notch length was changed from 8 mm to 15 mm in 0.2 mm step. The reflection coefficient regarding 500 kHz as a function of the notch length was shown in **Fig. 4**. The dots and line in Fig. 4 indicate the experimental results and the calculated values, respectively.

In the SI test, the notch length increased from 3 mm to 6 mm in 0.1 mm step while the notch depth increased simultaneously from 0.6 mm to 1.2 mm in 0.02 mm step. **Figure 5** shows the reflection coefficient of the SI test regarding 500 kHz as a function of the notch length.

The simple mathematical model described herein agreed fairly well with the two types of the experimental results.

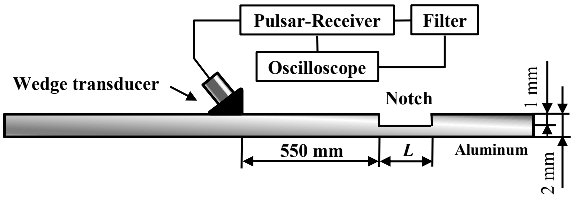

Fig. 3 Experimental setup.

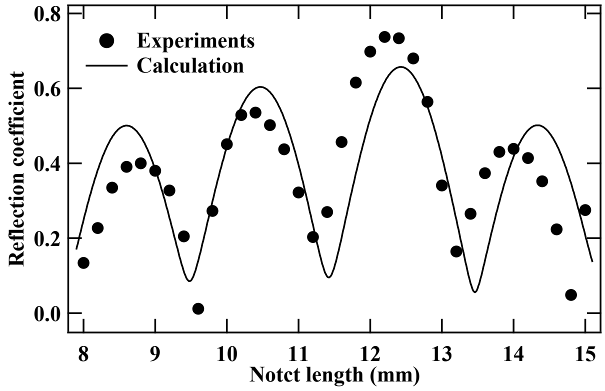

Fig. 4 Experimental results of the LI test in 500 kHz.

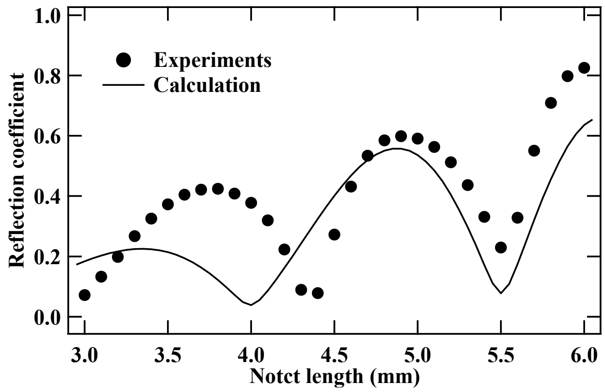

Fig. 5 Experimental results of the SI test in 500 kHz.

4. Conclusion

In this paper, a simple mathematical model at a two dimensional rectangular notch regarding the Lamb wave in the lowest frequency region was presented and verified. The simple mathematical model proposed herein clarified the primary reason of the reflection phenomena, that is, the interference among the several wave packets generated at the initial and final ends of the two dimensional rectangular notch took a dominant role to determine the reflection coefficient regarding A_0 mode Lamb wave.

References

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