1P3-2

Modified formula for electromechanical coupling coefficient using resonance--antiresonance measurement with a kind of nonlinear effect

共振反共振測定による電気機械結合係数の算出公式の修正 〜 一種の非線形形効果がある場合

Michio Ohki (National Defense Academy, Japan)
大木道生（防大）

1. Introduction

The electromechanical coupling coefficient, k^2, can be estimated with the measurement of the resonance and anti-resonance frequencies observed from the electrical port. It is also related with lumped-parameter elements of the electrical equivalent circuit as

$$k^2 = \frac{C}{C_0} \quad \text{for longitudinal (L-) effect,} \tag{1}$$

$$k^2 = \frac{C}{C + C_0} \quad \text{for transverse (T-) effect,} \tag{2}$$

where C_0 and C are the dielectric capacitance and the elastic equivalent capacitance including the infleunce of all modes, respectively. Equations (1) and (2) are obtained from considering the ratio of the converted energy to the total inputted energy at the situation of low frequency limit. In the L-effect, the C_0 and $-C_0$, a pair to C_0, constitute a kind of 'ideal gyrator,' which interchanges the behavior of electrical resonance and antiresonance frequencies with regard to the relationship between the elastic intrinsic state and the electromechanical coupled state in the T-effect.

However, an ideal gyrator cannot be realized using passive circuit elements, and eq. (1) implies that C cannot exceed C_0 in value, since $k^2 < 1$, although C and C_0 are physically independent from each other. Since nature behaves in a self-consistent manner, the unnatural inconsistency urges us to replace the ideal gyrator with a non-ideal one, which indicates some nonlinear characteristics, as shown later.

The point in this study lies in how the the unnatural inconsistency can be avoided and how the conventional formula for the estimation of k^2 from the resonance and antiresonance measurement is modified.

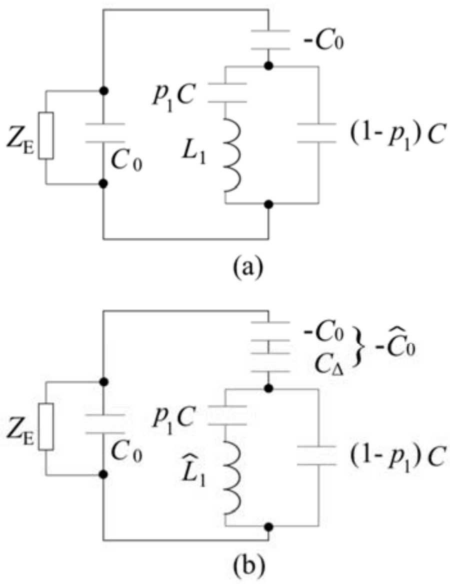

Fig. 1: (a) Conventional equivalent circuit of an electromechanical transduction system including an ideal gyrator at the lowest (1st) resonance, and (b) modified equivalent circuit including a non-ideal gyrator.

2. Review of estimation of k^2 using impedance in the conventional equivalent circuit

Figure 1(a) shows the conventional equivalent circuit for representing an electromechanical transduction system in the L-effect. The impedance around the circuit, Z, becomes $Z \to Z_{\text{open}}$ for $Z_{\text{E}} \to \infty$, and $Z \to Z_{\text{short}}$ for $Z_{\text{E}} \to 0$. We observe that $Z_{\text{open}} = 0$ at the lowest (first) angular antiresonance frequency $\omega = \omega_{\text{A1}} = 1/\sqrt{p_1 C L_1}$, and that $Z_{\text{short}} = 0$ at the lowest resonance one $\omega = \omega_{\text{R1}}$. Z_{short} is given by

$$Z_{\text{short}} = -\frac{1}{j\omega C_0} + \frac{1}{j\omega C}\frac{1 - \omega^2 p_1 C L_1}{1 - \omega^2 p_1 (1 - p_1) C L_1}, \tag{3}$$

and therefore,

$$\omega_{\mathrm{R1}}{}^2 = \left(1-\frac{C}{C_0}\right)\left(p_1CL_1\left(1-\frac{C}{C_0}\right)+p_1{}^2CL_1\frac{C}{C_0}\right)^{-1}, \quad (4)$$

which results in

$$\frac{k^2}{1-k^2} = \frac{1}{p_1}\frac{\omega_{\mathrm{A1}}{}^2-\omega_{\mathrm{R1}}{}^2}{\omega_{\mathrm{R1}}{}^2}, \quad (5)$$

since $k^2 = C/C_0$ and $\omega_{\mathrm{A1}} = 1/\sqrt{p_1CL_1}$. Equation (5) is well-known Marutake's formula. For a common boundary condition, $p_1 = 8/\pi^2$.

3. Modified estimation formula for k^2

In order to improve the characteristics involved in the ideal gyrator, a non-ideal gyrator is introduced by replacing $-1/C_0$ with $-1/\hat{C}_0$, as shown in Fig. 1(b), where

$$-1/\hat{C}_0 = -1/C_0 + 1/C_\Delta, \quad (6)$$

$$1/C_\Delta \to 0 \quad \text{for } C/C_0 \to 0. \quad (7)$$

By reconsidering the distribution of stored energy in the circuit, the relationship shown in eq. (1) is changed as

$$k^2 = \frac{C}{C_0}\times\frac{1}{1+\gamma}, \quad (8)$$

where

$$\gamma \equiv \frac{C}{C_\Delta} \quad (9)$$

is a parameter for the non-ideal gyrator, and the following series expansion is assumed for polynomial expansion coefficients a_m with integer m:

$$\gamma = \frac{C}{C_\Delta} = \sum_m a_m\left(\frac{C}{C_0}\right)^m. \quad (10)$$

The calculation results of the dependence of k^2 on C/C_0 for L-effect are shown in Fig. 2, where Fig. 2(a) shows the conventional result; that is, in the case of $\gamma = 0$, and Figs. 2(b) and 2(c) show two examples of improved results obtained from appropriate choices of a_m. The nonlinearity introduced in the circuit can remove the conventional unnatural restriction of $C/C_0 < 1$.

The addition of C_Δ also changes the elastic equivalent inductance element, as $L_1 \to \hat{L}_1$, since the elastic intrinsic resonance, or electrical antiresonance, should remain invariant. By setting $Z_{\mathrm{open}} = 0$ at

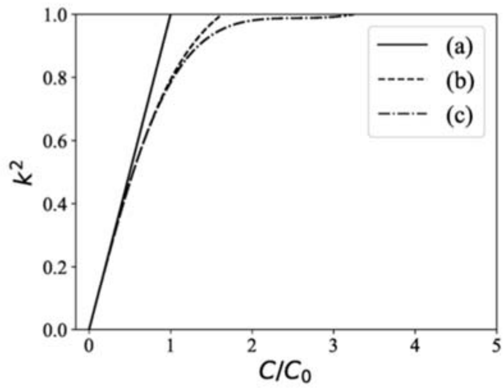

Fig. 2: Dependence of k^2 on C/C_0 for L-effect. (a) In the case of the conventional equivalent circuit. (b), (c) In the cases of the modified equivalent circuit. The choices for a_m's in (b) and (c) are different.

$\omega = \omega_{\mathrm{A1}} = 1/\sqrt{p_1CL_1}$, where

$$Z_{\mathrm{open}} = \frac{1}{j\omega C_\Delta} + \frac{1}{j\omega C}\frac{1-\omega^2p_1C\hat{L}_1}{1-\omega^2p_1(1-p_1)C\hat{L}_1}, \quad (11)$$

we obtain

$$\lambda_1 \equiv \frac{\hat{L}_1}{L_1} = \frac{1+\gamma}{1+(1-p_1)\gamma}. \quad (12)$$

By setting $Z_{\mathrm{short}} = 0$, where

$$Z_{\mathrm{short}} = -\frac{1}{j\omega C_0} + \frac{1}{j\omega C_\Delta} + \frac{1}{j\omega C}\frac{1-\omega^2p_1C\hat{L}_1}{1-\omega^2p_1(1-p_1)C\hat{L}_1}, \quad (13)$$

$\omega = \omega_{\mathrm{R1}}$ is calculated as

$$\omega_{\mathrm{R1}}{}^2 = \frac{1-\frac{C}{C_0}+\frac{C}{C_\Delta}}{p_1C\hat{L}_1\left(1-\frac{C}{C_0}+\frac{C}{C_\Delta}\right)+p_1^2C\hat{L}_1\left(\frac{C}{C_0}-\frac{C}{C_\Delta}\right)}. \quad (14)$$

By applying the following two relationships:

$$p_1C\hat{L}_1 = \frac{\hat{L}_1}{L_1}\cdot\frac{1}{\omega_{\mathrm{A1}}{}^2} = \frac{\lambda_1}{\omega_{\mathrm{A1}}{}^2}, \quad (15)$$

$$\frac{C}{C_0} - \frac{C}{C_\Delta} = k^2 + \gamma(k^2-1) \quad (16)$$

to eq. (14), a modified formula for estimating k^2 from a resonance and antiresonance measurement is derived:

$$\frac{k^2}{1-k^2} = \frac{1}{p_1}\frac{\omega_{\mathrm{A1}}{}^2-\lambda_1\omega_{\mathrm{R1}}{}^2}{\lambda_1\omega_{\mathrm{R1}}{}^2}(1+\gamma)+\gamma, \quad (17)$$

When $\gamma \to 0$, eq. (17) approaches to the conventional formula, eq. (5).