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A 3D Model for the Simulation of Narrowband Lamb Wave Excited by Modulating a Laser Beam with Michelson Interference Technique

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1. Introduction

Laser generated ultrasonic combined with optical detection method provides an effective approach to achieving noncontact and non-destructive evaluation. However, Lamb wave signal generated by laser is broadband and multi-mode, which causes difficulties to be utilized. To excite narrowband Lamb wave, researchers have modulated the laser beam with devices such as lenticular arrays [1], masks [2] and interferometers [3]. Different devices create various array patterns which produce different signals. In order to study the characteristics of the generated signals, the parameters of the laser array have been investigated. Previous studies [1-3] are concentrated on the two-dimensional analysis of the laser array whose element length is infinite and the intensity distribution is uniform. However, the actual situation is that the array length is finite and the distribution of the laser array is approximate to Gaussian distribution affected by the original laser beam, which is unable to be considered in the previous model. Also, the signal generated by some complex array profiles, such as incomplete laser array and irregular laser array cannot be predicted by the two-dimensional analysis.

In this paper, a three-dimensional laser array model for simulating the transient signal is created. This is achieved by dividing the laser array into point sources and superposing all the point source generated signals from the laser array. The transient signals at different distances are calculated by integral transform methods combined with numerical calculation methods. With this model, the complex profile of the laser array can be studied. More importantly, it greatly shortens the calculation time compared with the 3D analysis module of the FEM software. With the purpose of confirming the simulation model is a good representation of the actual situation, the experiment is implemented. The numerical result shows good agreement with the experimental result.

2. Theoretical and Numerical approach

In our model, the laser array is considered as an assembly of point source as shown in **Fig. 1**. The transient wave at the detection point is the superposition of the signals generated by all point sources of the laser array, and the amplitude of the point source generated signals is decided by the corresponding energy intensity of the laser array. Thus, the displacement at the detection point is obtained:

$$u(x,y,t)=\sum u_i\,(r_i,t)w(x_i,y_i), \qquad (1)$$

where r_i is the distance between the detection point (x,y) to the source point (x_i,y_i), $w(x_i,y_i)$ is the energy intensity of the laser array at point (x_i,y_i), and u_i is the transient waveform at a distance r_i away from the point source, calculated by integral transform combined with FFT technique.

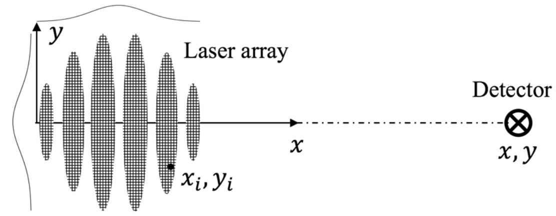

Fig. 1 Simulation model of the laser array.

With regard to the energy intensity w of the laser array modulated by the Michelson interference technique, which used in this paper, the distribution along x axis can be described as the Gaussian distribution of the original laser beam multiplied the $\cos^2$ pattern [4] distribution caused by the interference and the distribution along y axis is simply Gaussian distribution without $\cos^2$ pattern. **Fig. 2** shows a standard laser array model whose element spacing is 1.6mm and the element number is 6.

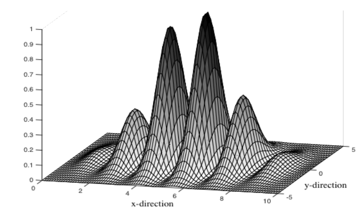

Fig. 2 Energy profile of the laser array

In order to calculate the transient waveform of the Lamb wave generated by different point sources, cylindrical coordinates are used, as shown in **Fig. 3**.

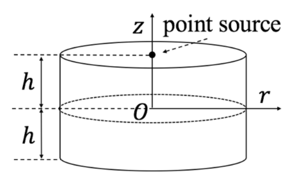

Fig. 3 Schematic of cylindrical coordinates for calculating the point source generated signal.

Considering a laser point source in the thermoelastic regime impacts on the upper surface of an aluminum plate, whose thickness is 2h, the stress boundary

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conditions of two surfaces for this case can be given as

$$\begin{aligned} &\sigma_{zz}(r,h,t)=0 \qquad \sigma_{zr}(r,h,t)=\delta(r)s(t) \\ &\sigma_{zz}(r,-h,t)=0 \qquad \sigma_{zr}(r,-h,t)=0 \end{aligned} \tag{2}$$

where $s(t)$ is the laser pulse function which can be represented as $s(t)=(t/\tau^2)e^{-t/\tau}$ [5] and $\delta(r)$ is the Dirac function.

Applying the Fourier transform and the Hankel transform [6] on r and t, respectively, to the stresses and displacements, and then substituting the solution of potentials into them, we obtain the transformed equation of the displacements and stresses.

The inverse Fourier transform and Hankel transform are used to calculate the transient response of the displacement of the surface as shown in follow

$$u(r,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{0}^{+\infty}\hat{u}(\xi,\omega)\xi J_0(r\xi)d\xi e^{i\omega t}d\omega, \tag{3}$$

where $\hat{u}(\xi,\omega)$ is the transformed equation of the out-of-plane displacement and J_0 is the zeroth order Bessel function. Eq.(3) can be calculated numerically by using FFT technique [7].

In our calculation, the distance between the point sources used to simulate the laser array is 0.1mm and the laser array profile is the same with the array modulated in the experiment. Parameters of the aluminum plate are given in **Table I**.

Table I Paremeters of the aluminum plate

	Density /g·cm^{-3}	C_L /km·s^{-1}	C_T /km·s^{-1}	Thickness /mm
Al	2.7	6.3	3.1	0.5

3. Experimental setups

The experimental setups are shown in **Fig. 4**. For generating Lamb wave, a 532 nm Q-switch Nd: YAG laser is used. A concave lens and a convex lens are used to expand and collimate the original beam to 10mm. Two mirrors and a beam splitter are used to build a Michelson interferometer for modulating the expanded laser beam. A rotating platform is used to change the angle between two mirror, thus change the spacing of the laser array. A broadband laser interferometer is used to receive the out-of-plane displacement. The thermal paper is used to record the profile of the laser beam after every shot.

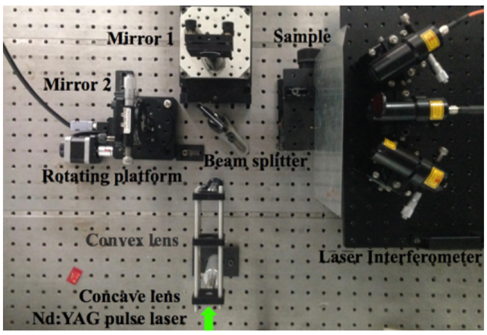

Fig. 4 Experimental setups.

4. Results and Discussion

A 1.6mm spacing laser array modulated by the Michelson interferometer is used to generate narrowband Lamb wave whose modes propagate at different velocities, thus each mode can be clearly recognized in the time domain. **Fig. 5** shows a comparison between the experimental and the numerical results implemented on a 0.5mm aluminum plate. By the modulated laser beam, the narrowband A0, S0, and A1 Lamb waves are excited successfully and the simulation result agrees well with the experimental result. It can be confirmed that the 3D laser array model can perfectly simulate the actual generated signal.

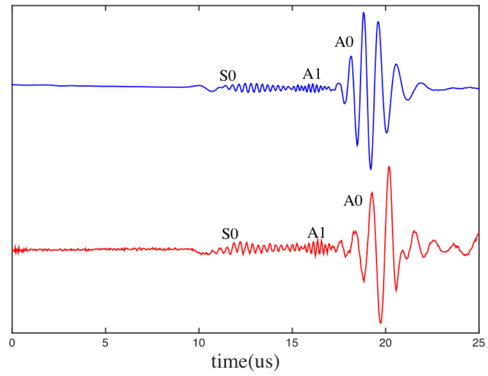

Fig. 5 Experimental signal (red) and theoretical signal (blue) of the 1.6mm spacing laser array

With this model, signals generated by different laser array can be easily simulated by just changing the energy distribution coefficient w of the laser profile. Various laser array profiles can be studied with this model, especially some irregular distribution caused by unparallel interference beam or special designed devices. Influence of laser array parameters on generation of narrowband Lamb wave excited by modulated laser beam will be studied by using this 3D laser array model in the future.

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