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Time evolution analysis of Coherent Population Trapping resonance for atomic clock chip

原子時計チップ化のための Coherent Population Trapping 共鳴の時間発展解析

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1. Introduction

Coherent population trapping (CPT) resonance plays an important role that the fully optical interrogation of the clock transition of atoms and contributes to reducing the cost, size, and power consumption of atomic clock systems[1]. To design atomic clocks, the time response of the CPT resonance under various perturbations must be modeled elaborately. An algorithm to calculate the time response of CPT resonance with periodic boundary condition has been proposed for analyzing pulsed CPT resonance [2,3]. In this algorithm, the time is divided into short intervals, and the response is calculated for each section. By increasing the number of divisions, it can be applied to the analysis of CPT resonance under any perturbation. However, this method suffers from quantization and aliasing errors due to the time division. Further decreasing the time interval to relieve these problems greatly increases the calculation cost.

To overcome this trade-off relation, we attempted to apply the Galerkin method using a Fourier series as a basis function (Galerkin spectral method) to the analysis of CPT resonance. The Galerkin spectral method has been widely used in the numerical analysis of piezoelectric mechanical resonators such as quartz to reduce the calculation cost[4]. Unlike in previous works, a continuous function can be applied to calculate the time response in the new algorithm. Therefore, the calculation cost can be substantially reduced while maintaining high precision compared with the conventional algorithm.

2. Theoretical model

Figure 1 (a) shows the excitation scheme for the ^{133}Cs-D$_1$ line. In the CPT phenomenon, two ground states of $6^2S_{1/2}$ are coupled simultaneously to a common excited state of $6^2P_{1/2}$. To calculate the behavior of the CPT resonance, a closed Λ-type three-level model is often used as shown in Fig. 1 (b). Here, $|1\rangle$ and $|2\rangle$ correspond to the two ground states $|F = 3, m_F = 0\rangle$ and $|F=4, m_F = 0\rangle$ in the $6^2S_{1/2}$ state, respectively, and $|3\rangle$ corresponds to a state in $6^2P_{1/2}$.

The frequency detuning of a microwave transition (δ_p - δ_c) must be modulated for the synchronous detection of the clock transition and is

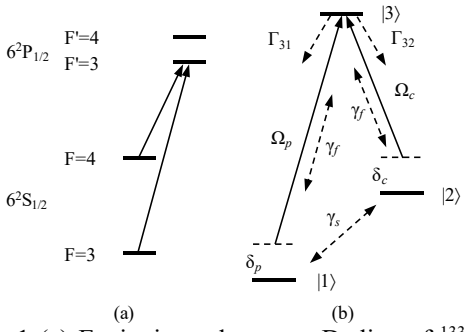

Fig. 1 (a) Excitation scheme on D_1 line of ^{133}Cs. (b) Closed Λ-type three-level model used to calculate the CPT resonance.

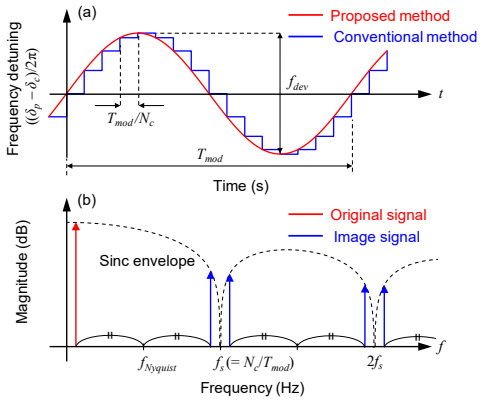

Fig. 2 Interrogation scheme in (a) time domain and (b) frequency domain: red and blue line shows the interrogation curve with proposed and conventional method, respectively.

expressed as follows:

$$\delta_p - \delta_c = \frac{2\pi f_{dev}}{2}\sin(\omega_{mod}t), \qquad (1)$$

where ω_{mod} is the angular frequency of the modulation ($\omega_{mod} = 2\pi/T_{mod}$) and f_{dev} is the frequency deviation. In conventional method, the time division generates a quantization error and image signals as shown in Fig. 2. The image signals are difficult to cut off by the sinc filter if number of divisions N_c is not sufficiently large. Therefore, we usually face a trade-off between the precision and the calculation cost.

3. Calculation method

The absorption spectrum of CPT resonance is evaluated using density matrix. The density matrix ρ is governed by the quantum Liouville equation,

$$\frac{\partial\rho}{\partial t} = \frac{i}{\hbar}[\rho, H] + R\rho, \qquad (2)$$

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where H is the Hamiltonian matrix, which includes the intensity and the frequency detuning. Rρ is the relaxation matrix, which represents the collision between the buffer gas and Cs atoms. Eq. (2) is rewritten to be

$$\frac{\partial}{\partial t}|\rho) = \tilde{H}|\rho), \quad (3)$$

where $|\rho)$ is a vector that consists of 9 elements of ρ. $\tilde{H}$ is a matrix with a size of 9 × 9 including the relaxation terms. $\tilde{H}$ can be decomposed from the time dependence as

$$\tilde{H} = \tilde{H}_0 + \tilde{H}_{int} f(t) \quad (4)$$

where $f(t)$ is any function of time and $\tilde{H}_{int}$ is independent of the time and includes the parameter f_{dev}. Substituting Eq. (4) into Eq. (3), we obtain

$$\left(\frac{\partial}{\partial t} - \tilde{H}_0 - \tilde{H}_{int} f(t)\right)|\rho) = \vec{0}. \quad (5)$$

Subsequently, we apply the Galerkin spectral method to solve Eq. (5) for $|\rho)$. Using the Fourier transform, $f(t)$ is expressed by

$$f(t) = \sum_{k=1}^{N_p}(a_{2k-1}\cos(k\omega_{mod}t) + a_{2k}\sin(k\omega_{mod}t)). \quad (6)$$

Similarly, the density matrix is

$$|\rho) \approx \vec{c}_0 + \sum_{k=1}^{N_p}(\vec{c}_{2k-1}\cos(k\omega_{mod}t) + \vec{c}_{2k}\sin(k\omega_{mod}t)) = C\varphi, \quad (7)$$

where N_p is the maximum harmonic number and is an important factor in determining the time resolution. $\vec{c}$s are the fitting coefficients of each frequency components. C is a 9 × ($2N_p$ + 1) matrix composed of $\vec{c}_0$, $\vec{c}_1 \cdots \vec{c}_{2N_p}$. φ is the shape function

$$\varphi = (1, \sin(\omega_{mod}t), \cos(\omega_{mod}t), \cdots \sin(2N_p\omega_{mod}t), \cos(2N_p\omega_{mod}t))^T. \quad (8)$$

We obtain the following relation using the weighted residual method:

$$\int_0^{T_{mod}} \varphi \left[\left(\frac{\partial}{\partial t} - \tilde{H}_0 - \tilde{H}_{int} f(t)\right)|\rho)\right]^T dt = O, \quad (9)$$

where, O is zero matrix with a size of ($2N_p$ + 1) × 9. Substituting Eqs. (6) and (7) into Eq. (9), we obtain

$$MC = O, \quad (10)$$

where M is the overall matrix with a size of ($2N_p$ + 1) × 9. Solving Eq. (10) for C, $|\rho)$ can be determined under the standardized condition ($\sum \rho_{i,i}$ = 1).

4. Results

The time response of CPT resonance with f_{dev} = 10 kHz, which reaches the off-resonance, is shown in Fig. 3. The other parameters are set as Table I. In this case, the time response was considerably different from the sinusoidal response. It is difficult to obtain high precision with a small N_p in the proposed method. However, from viewpoint of the convergence speed, the proposed method gives good performance as shown in Fig. 4, which shows the L_2 relative error norm as functions of N_c and N_p. Red open circles and blue filled circles are the relative errors calculated using the proposed method and conventional method, respectively. In both methods, the calculation result of the proposed method at N_p = 2048 is used as the true value to calculate the relative errors. The relative calculation time as a function of N_c and N_p is shown in Fig. 4. The vertical axis of this figure was normalized by the calculation time at N_p = 128. Comparing the relative error at relative calculation time of 1, the difference between the conventional and proposed method reaches more than 5 orders of magnitude. It was confirmed that the proposed method is extremely effective to reduce the calculation cost and improve the calculation precision.

Table I Setting Parameters

Rabi frequency Ω_p, Ω_c	1 MHz
Ground-state relaxation rate γ_s	100 MHz
Total emission rate $\Gamma_3 = \Gamma_{31} + \Gamma_{32}$	720 MHz
Modulation frequency f_{mod}	100 Hz

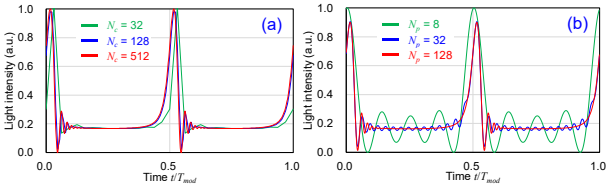

Fig. 3 Time response of CPT resonance under different N_c and N_p (f_{dev} = 10 kHz): (a) conventional method, (b) proposed method.

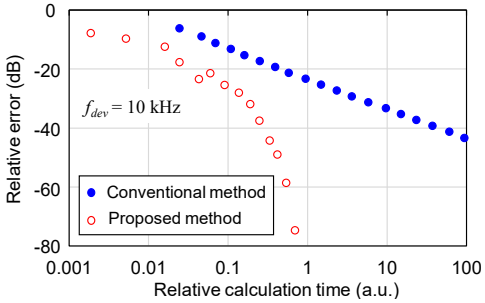

Fig. 4 Relative errors with conventional method and proposed method as a function of (a) N_c and N_p and (b) relative calculation time: f_{dev} = 10 kHz.

4. Conclusions

We propose the use of the Galerkin spectral method to simulate the time response of the CPT resonance. To optimize the performance of CPT clocks, it is necessary to simulate the time response of the CPT resonance under any external perturbations. In this work, it was confirmed that the proposed method shows comparable precision to the conventional method despite its low calculation cost.

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