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Measurement for Transfer Function of Delay Line Using Acoustical Frequency Comb

音響周波数コムを用いる遅延線の伝達関数測定

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1. Introduction

Recently, various researches on optical freqency comb are actively carried out. Optical freqency comb has a spectrum arranged at equal interval. Therefore, it enables accurate freqency measurement and more precious gas detection[1]. In this paper, we apply the theory of optical freqency comb to acoustics, and we call it '**acoustical frequency comb**'. Comb signal has a wide-band and regular intervals spectrum. We expect that comb signal can measure transfer function of delay line.

2. Theory

Figure 1 shows the configuration of linear time-invariant system. An input to system is $x(t)$ and an output is $y(t)$. The system impulse response is $h(t)$. Then, the transfer function $H(f)$ is given as

$$H(f) = \frac{Y(f)}{X(f)} \tag{1}$$

where, $X(f)$ and $Y(f)$ are Fourier transforms of $x(t)$ and $y(t)$.

Generally, the signal $x(t)$ used for transfer function measurement is expected to have wide-band frequency spectrum and high energy such as time stretched pulse (TSP) [2]. If the high-intensity sound source is required for the measurement, the sources tend to have narrow-band characteristics. In this situation, the sound source is not suitable for generating wide-band signals like TSP. Rapid amplitude modulations by acoustical means can widen the bandwidth of the signal even if it has been generated by narrow-band devices. Thus, we focused on acoustical frequency comb, which is periodically amplitude modulated acoustic signal.

Figure 2 shows how to calculate transfer function with frequency comb signal. Fig. 2(a) is the time waveform of comb signal $x(t)$. Comb signal consists of multiple periodical Gaussian modulated cosine (GMC) waves. Each GMC wave has center frequency $(f_{c1}, f_{c2}, f_{c3}, \ldots)$. The comb signal $x(t)$ is given as

$$x'(t) = \sum_{k=1}^{K} A e^{-B\left(t-\frac{kT}{K}\right)^2} \cos\left\{2\pi f_{ck}\left(t-\frac{kT}{K}\right)\right\}$$

$$x(t) = \sum_{m=1}^{M} x'(t-mT) \tag{2}$$

where T is repeated interval of each GMC wave, A is amplitude of signal, B is Gaussian attenuation term, K is number of the center frequency, and M is repeat count of GMC waves.

In Fig. 2(a), the comb signal consists of four GMC waves (K=4). Fig. 2 (b) shows Fourier transform of input and output comb signals. Comb signal has a spectrum that is arranged at equal intervals every $1/T$ shown in the lower Fig. 2 (b). Then, discrete frequency characteristic H is given as

$$H\left(\frac{i}{T}\right) = \frac{Y_i}{X_i} \quad (i = 1,2,\ldots,\mathrm{n},\ldots) \tag{3}$$

where, X_i and Y_i are I-th mode comb of $X(f)$ and $Y(f)$. As a calculation result, we obtain discrete frequency characteristic of amplitude and phase.

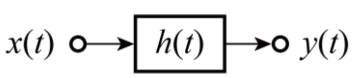

Fig. 1 Linear time-invariant system I/O relationship.

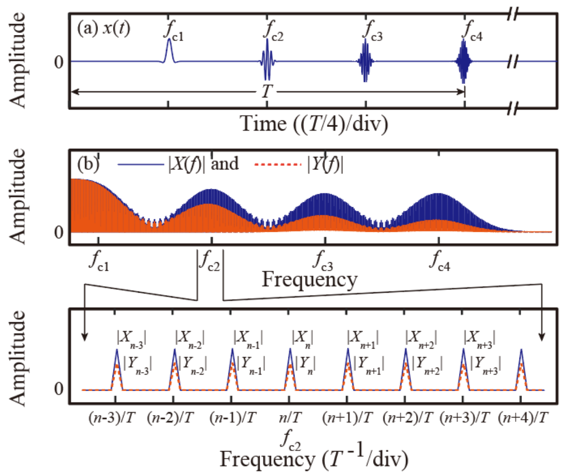

Fig. 2 Method of calculate transfer function using comb signal.

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3. Experiment

To confirm the feasibility of transfer function measurement using frequency comb signal, we conducted two experiments.

At first experiment, we measured the first order LPF's transfer function with the comb signal. LPF's cutoff frequency was 10 kHz. The measurement system consists of DAC (NI USB-6212), ADC (NI USB-6212), 1.5 kΩ metal resistor and 0.01 μf ceramic capacitor. At first, we generate the frequency comb signal $x(t)$ with a computer. Second, we record the generated signal by the DAC and detect the LPF output signal $y(t)$ with the ADC and the computer. In this experiment, comb signal $x(t)$ consisted of ten GMC waves (K=10). Each center frequency f_c was defined at 10 kHz intervals from 2.5 kHz to 92.5 kHz (f_{c1}=2.5 kHz, f_{c2}=12.5 kHz, …, f_{c10}=92.5 kHz). And, these functions cycle T was 0.01 s. Thus, this comb signal had a spectrum at 100 Hz (=1/T) intervals. Sampling rate was 400 kHz. **Figure 3** shows LPF's transfer function measurement result. As a conventional method, we measured the transfer function with spectrum analyzer (dashed line plot). On the other hand, the proposed method using comb signal was calculated according to equation 3 (line plot). We found that the method with spectrum analyzer and the proposed method using the comb signal almost agreed with each other. However, in the part such as 38 kHz and 88 kHz, the value was out of order. This may cause the frequency spectrum of the input signal is small.

At second experiment, we measured the acoustic delay line transfer function $H(f)$ including electronic circuits shown in **Fig.4**. This measurement system consists of DAC, ADC, two operational amplifiers (LM386), a speaker (FOSTEX P650K) in the speaker box (P650E) and a microphone (C9767). The length between the speaker and the microphone was 0.3 m. At first, we record the generated signal $x(t)$ to the DAC and play the sound with the speaker. Next, the microphone measures the sound and detect the output signal $y(t)$ with the ADC and the computer. In this experiment, comb signal consisted of five GMC waves (K=5), and each wave's center frequency f_c was defined at 10 kHz intervals from 2.5 kHz to 42.5 kHz (f_{c1}=2.5 kHz, f_{c2}=12.5kHz, …, f_{c5}=42.5 kHz). And, these functions cycle T was 0.02 [s]. Thus, this comb signal had a spectrum at 50 Hz (=1/T) intervals. and the amplitude A was 0.3 V. In addition, we applied the chirp signal under same condition and calculated the frequency characteristics of amplitude and phase. The chirp signal was swept linearly from 100 Hz to 40 kHz. **Figure 5** shows space transfer function measurement result. We found that the characteristic of amplitude result using comb signal and the result using chirp signal almost agreed with each other. And we found that the desired result was also obtained for the phase.

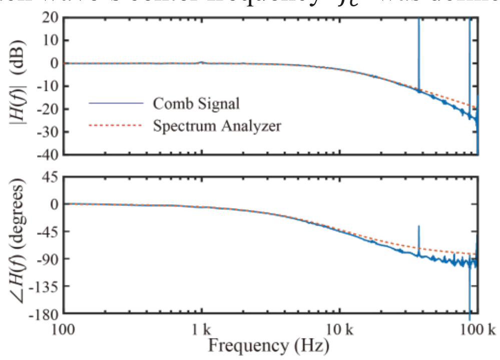

Fig. 3 Measurement result of LPF's frequency characteristics.

4. Conclusion

We proposed a method of using acoustical comb signal as a method of frequency characteristics measurement and confirmed the feasibility through the experiment. From the result of experiment, we found that comb signal can measure transfer function of delay line.

We are considering using acoustical frequency comb for other measurement object. As one example, transmission of sound in the humidity space is frequency dependent. Therefore, we are considering optical comb signal apply humidity measurement with narrow-band device.

References

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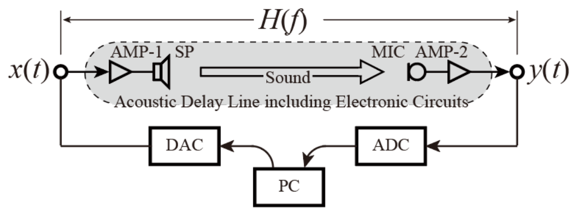

Fig. 4 Measurement system of space transfer function.

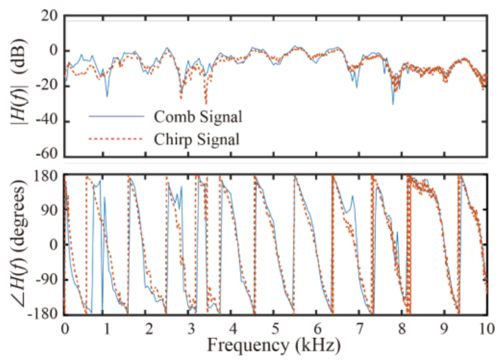

Fig. 5 Measurement result of SP-MIC frequency characteristics.