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Modelling of the nonlinear vibration of 33 effect transducer with transfer matrix

伝達マトリックスを用いた 33 効果振動子の非線形振動のモデル化

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1. Introduction

Piezoelectric transducers are used in various ultrasonic devices and they are driven under high power condition. Nonlinear effects such as “jumping phenomena” in current, hysteresis of admittance curve and saturation of the vibration velocity of piezoelectric transducer under high power driving are well-known[1]. It is originated from the mechanical nonlinearity rather than the dielectric and piezoelectric nonlinearities[2]. It indicates that large strain magnifies the influence of the higher order elasticity. Performance of piezoelectric transducer is usually evaluated using the piezoelectric constant, electromechanical coupling factor and mechanical quality factor; however, they are based on the linear piezoelectric equation. Therefore, piezoelectric transducer used in the ultrasonic devices cannot be evaluated and selected appropreatelly. In addition, FEM analysis, which is frequentlly used to design ultrasonic devices, doesn't consider the higher order elasticity. To design high power ulatrasonic devices, it is neccesarry to evaluate higher order elastic constant of piezoelectric transducer.

Our research group has already developed measurement method of higher order elastic constant for piezoelectric 31 effect transducer using nonlienar *LCR* equivallent circuit and transfer matrix[3]. On the other hand, piezoelectric 33 effect transducer are often used in actual ultrasonic devices. Additionally, higher order elastic constants for various direction must be considered to realize the nonlinear calculation in FEM analysis. Therefore, in this study, we developed the measurement method of higher order elastic constant of piezoelectric 33 effect transducer. From the admittance measurement, higher order elastic constants were determined by transfer matrix calculation. Hard-type PZT (Fuji-ceramics C203) and soft-type PZT (Fuji-ceramics C6) transducer were investigated and evaluated.

2. Nonlinear model

2.1 Higher order elasticity

Piezoelectric equation of 31 effect with higher order elasticity is expressed as equations (1)-(2)[2]:

$$T_3 = \overline{c_{33}^E} S_3 + c_{33(3)}^E S_3^3 - \overline{e_{33}} E_3 \tag{1}$$

$$D_3 = \overline{e_{33}} S_1 + \overline{\varepsilon_{33}^S} E_3 \tag{2}$$

where T_3, S_3, E_3, D_3 are strain, stress, electric field and electric flux density; $\overline{c_{33}^E} = \frac{1}{s_{33}^E}$, $c_{33(3)}^E$, $\overline{e_{33}} = \frac{d_{33}}{s_{33}^E}$, $\overline{\varepsilon_{33}^S} = \varepsilon_{33}^T - \frac{d_{33}^2}{s_{11}^E}$ are linear elastic constant, cubic elastic constant, piezoelectric constant and dielectric constant. As indicated, only the higher term $c_{33(3)}^E S_3^3$ is considered and other higher terms are not taken into account because the even-ordered terms don’t affect the signal with driving frequency and the influence of larger than 5th order terms is negligible. In this research, cubic elastic constant $c_{33(3)}^E$ was measured. The parameters, $\overline{c_{33}^E}$ and $c_{33(3)}^E$, were treated as complex number as equation (3)-(4):

$$\overline{c_{33}^E} = \mathrm{Re}\left(\overline{c_{33}^E}\right) + j\mathrm{Im}\left(\overline{c_{33}^E}\right) \tag{3}$$

$$c_{33(3)}^E = \mathrm{Re}\left(c_{33(3)}^E\right) + j\mathrm{Im}\left(c_{33(3)}^E\right) \tag{4}$$

2.2 Nonlinear transfer matrix

From the nonlinear piezoelectric equation (1)-(2), wave equation is expressed as equation (5):

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \tag{5}$$

where displacement u is expressed as :

$$u(x,t) = u_x e^{j\omega t} \tag{6}$$

and sound velocity $c = \sqrt{\frac{c'_{33}}{\rho}}$. The parameter c'_{33} is nonlinear elastic constant which depends on internal stress. It is expressed as equation (7):

$$c'_{33}(x) = \overline{c_{33}^D} + \frac{3}{4} c_{33(3)}^E \left(\frac{\partial u_x}{\partial x}\right)^2 \tag{7}$$

As shown in equation (5), nonlinear elastic constant $c'_{33}(x)$ depends on strain $\frac{\partial u_x}{\partial x}$. Therefore, transfer matrix is used to calculate strain distribution. Transfer matrix is expressed as equation (8):

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$$\begin{pmatrix} F_n \\ v_n \\ D_3 \end{pmatrix} = \begin{pmatrix} -\frac{a}{b} & \frac{-a^2+b^2}{b} & \frac{A(a+b)}{b} \\ -\frac{1}{b} & -\frac{a}{b} & \frac{A}{b} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} F_{n+1} \\ v_{n+1} \\ D_3 \end{pmatrix} \qquad (8)$$

where $a = jSZ\left\{\tan\left(\frac{kl}{2}\right) - \frac{1}{\sin(kl)}\right\}$, $b = \frac{jSZ}{\sin(kl)}$, wave number $k = \frac{\omega}{c}$, force factor $A = \frac{wh\overline{e_{33}}}{\overline{\varepsilon_{33}^S}}$, acoustic impedance $Z = \rho c$, cross-sectional area $S = wh$ and w, h are width, height of the transducer. F_n, F_{n+1} are force and v_n, v_{n+1} are velocity at each surface. In the case of piezoelectric 33 effect transducer, electric field depends on internal strain because electric field and strain are parallel. On the other hand, electric flux density D_3 doesn't depend on position. The parameter D_3 is obtained as following equation

$$D_3 = \frac{\overline{\varepsilon_{33}^S}}{l} V + \frac{\overline{e_{33}}}{j\omega l}(v_2 - v_1), \qquad (9)$$

where V is input voltage, l is total length of the transducer and v_1, v_2 are tip velocity of the transducer. Current I is calculated from D_3 as equation (10):

$$I = \frac{d}{dt}\int_S D_3\, dS = j\omega S D_3 \qquad (10)$$

3. Measurement

In this research, admittance curve was measured and used to obtain higher order elastic constant. Hard-type PZT (Fuji-ceramics C203) and soft-type PZT (Fuji-ceramics C6) transducer were measured. The dimensions of the transducers were 2 mm × 3 mm × 10 mm and polarization direction was aligned to 10 mm direction. Frequency response analyzer (NF FRA5097) and power amplifier (NF 4010) were used to measure the admittance curve under low voltage (10 Vpp) and high voltage (100 Vpp). From measured results, curve fitting was conducted using the nonlinear transfer matrix for obtaining higher order elastic constant.

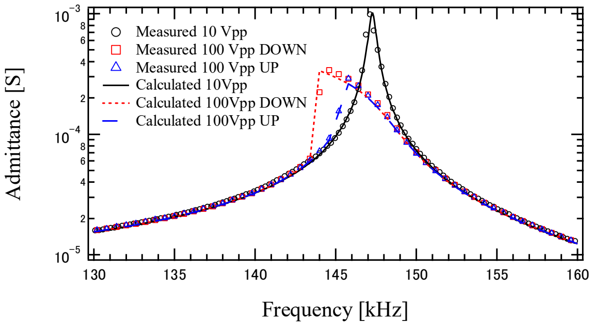

Fig. 1 Measured and fitted admittance curve of hard-type PZT (C203).

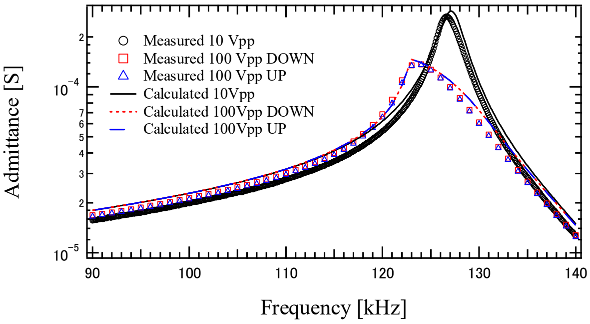

Fig. 2 Measured and fitted admittance curve of soft-type PZT (C6)

4. Measurement

Figure 1 and 2 show measured and fitted admittance curve of hard-type (C203) and soft-type PZT (C6). Fitting result showed good agreement with measurement result. Table 1 and 2 show higher order elastic constant obtained from curve fitting. 33 effect and 31 effect showed same tendency that absolute value of higher order elastic constant of soft-type PZT were larger than hard-type PZT.

5. Conclusion

In this research, measurement of higher order elastic constant of 33 effect transducer was examined. It was revealed that hard-type PZT had smaller higher order elasticity than soft-type PZT.

Table 1 Higher order elastic constant of 33 effect

	$\mathrm{Re}\left(c_{33(3)}^E\right)$ [N/m^2]	$\mathrm{Im}\left(c_{33(3)}^E\right)$ [N/m^2]
C-203	-1.2×10^{17}	7.0×10^{15}
C-6	-7.7×10^{17}	2.3×10^{17}

Table 2 Higher order elastic constant of 31 effect

	$\mathrm{Re}\left(c_{11(3)}^E\right)$ [N/m^2]	$\mathrm{Im}\left(c_{11(3)}^E\right)$ [N/m^2]
C-203	-1.3×10^{17}	8.5×10^{15}
C-6	-2.5×10^{18}	8.0×10^{17}

References

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