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L1-norm Measurement of Medical Ultrasound Signal Using Overcomplete Dictionaries

過完備辞書を用いた生体超音波信号の L1 ノルム測定

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1. Introduction

Recently, compressive sensing (CS) has been attracting in many fields. This method can reconstruct signals using a small number of measurements by assuming sparsity of data in a suitable bases [1]. It has already been applied to magnetic resonance imaging (MRI) and X-ray computed tomography (CT), enabling high-speed projection while maintaining image quality. In medical ultrasound images, researches have investigated to reduce the number of elements and to improve the image quality without increasing the number of measured data [2,3].

In CS, if the target is non-sparse, it is necessary to transform bases into others to obtain a sparse representation of the observation signal. Some signals can sparsify using Fourier transform (FT), discrete cosine transform (DCT), wavelet transform and so on [4]. However, some signals like ultrasound echo from human body, cannot be transformed into sparse using these traditional bases.

In this paper, the combination of DCT and wavelet transform was used. By using redundant bases, it is possible to make coefficients containing many zero components. If length of observed signal data is smaller than that of the bases, this base is called overcomplete dictionary (OD) [5,6,7]. To evaluate the sparsity, the L0 and L1 norms of the coefficients obtained using several sets of bases were measured.

2. Method

In this study, L1 norms of the coefficients with various sparse matrix were measured.

CS method, assuming a signal $x \in \mathbb{R}^N$ is sparse, it can be recovered from its measurements $y \in \mathbb{R}^M$ $(M < N)$ with high probability [1,8].

$$\boldsymbol{y} = \boldsymbol{\Phi x} \tag{1}$$

where $\Phi \in \mathbb{R}^{N\times N}$ is random with average of zero and σ of $1/\sqrt{M}$, and called the measurement matrix. Even if the signal is non-spars, it can be represented sparsely in a certain basis.

$$\boldsymbol{x} = \boldsymbol{\Psi v} \tag{2}$$

where $v \in \mathbb{R}^P$is sparse representation of x. Its L0 norm s, which is the number of nonzero entries, become smaller than M. In this study, a combination of DCT and Haar bases was prepared as the sparse representation $\Psi \in \mathbb{R}^{N\times P}$, and it is called the overcomplete dictionary. Subsequently, substituting eq. (2) into eq. (1), we can obtain

$$\boldsymbol{y} = \boldsymbol{\Phi\Psi v} = \boldsymbol{Av} \tag{3}$$

where $A \in \mathbb{R}^{M\times P}$. Observed signal y must include additive noise, eq. (3) can be recast as

$$\boldsymbol{y} = \boldsymbol{Av} + \boldsymbol{e} \tag{4}$$

where $e \in \mathbb{R}^N$ represents noise.
The sparsest solution $\hat{v}$ of eq. (4) is obtained by L1 optimization expressed as below

$$\hat{\boldsymbol{v}} = \mathbf{min}\|\boldsymbol{v}\|_1 \ \mathbf{s.t.}\ \|\boldsymbol{y} - \boldsymbol{Av}\|_2^2 \le \boldsymbol{e}. \tag{5}$$

When sparse coefficient v is obtained in eq. (5), solution x is derived using eq. (2).

In this study, several types of sparse matrix for Ψ in eq. (3) were prepared and evaluated: the discrete cosine transformation (DCT) basis, the Haar basis, and an overcomplete dictionary using the DCT and the Haar, which is called DH basis ($\Psi_{DH} \in \mathbb{R}^{N\times 2N}$).

Since both the DCT and the Haar bases are regular matrices, the coefficients v can be obtained by taking the inverse matrix of each base matrix.

$$\boldsymbol{v_{DCT}} = \boldsymbol{\Psi_{DCT}^{-1} x} \tag{6}$$

$$\boldsymbol{v_{Haar}} = \boldsymbol{\Psi_{Haar}^{-1} x}. \tag{7}$$

While, when using the DH basis, eq. (2) becomes underdetermined. To solve this problem, least absolute shrinkage and selection operator (lasso) [9] algorithm was used as below.

$$\mathbf{min}\|\boldsymbol{y_{DH}}\|_1 \ \mathbf{s.t.}\ \boldsymbol{y_{DH}} = \boldsymbol{\Psi_{DH} v_{DH}} \tag{8}$$

L1 norm l_1 and L0 norm l_0 of the coefficients can be obtained by the following eqs.

$$\boldsymbol{l_1} = \sum |\boldsymbol{y_i}| \tag{9}$$

$$\boldsymbol{l_0} = \sum \boldsymbol{\delta}(\boldsymbol{y_i}) \tag{10}$$

$$\boldsymbol{\delta}(\boldsymbol{a}) = \begin{cases} \mathbf{1}\ (\boldsymbol{a} \neq \mathbf{0}) \\ \mathbf{0}\ (\boldsymbol{a} = \mathbf{0}) \end{cases}. \tag{11}$$

The smaller L1 norm of the coefficients is, sparser v is. Therefore, it is considered to be an appropriate basis for x.

Our goal is to reconstruct the fine signal from fewer measurements. In this study, the measurement

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signal was prepared by decimating the fine signal. Finding the proper basis for medical ultrasound signals to be sparse representable will allow the application of CS.

3. Results

To obtain the measurement signal y, a numerical simulation FIELD II was used. In this simulation, a phantom for a left kidney is scanned with a 7 MHz 128 element convex phased array transducer, and 128 lines with 0.7 degrees between lines are obtained. An example of the measured signals is shown in **Fig. 1**. This signal was transformed into DCT, Haar, and DH, separately. The calculated coefficients from the signal of Fig. 1 are shown in **Figs. 2, 3,** and **4**, respectively. Averages of L1/L0-norms with various bases are listed in **Table. I**. It is clear that the L1 norm of DH basis was 3.3 and significantly smaller than those of DCT and Haar.

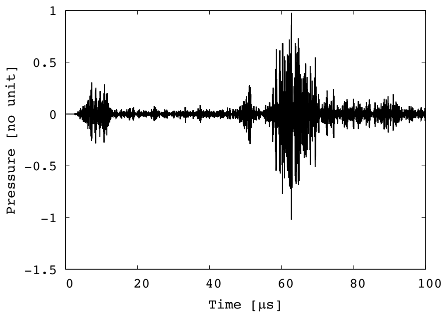

Fig. 1 RF signal from kidney in time domain.

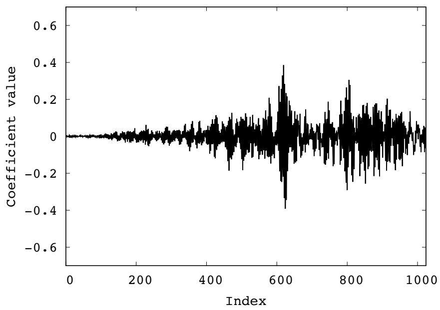

Fig. 2 Coefficients of DCT (N=1024)

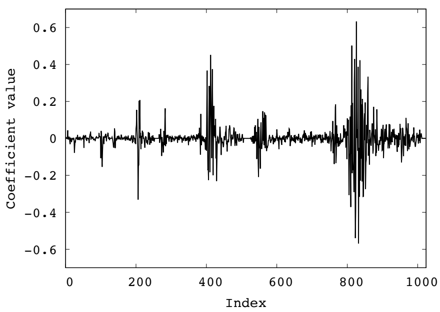

Fig. 3 Coefficients of Haar (N=1024)

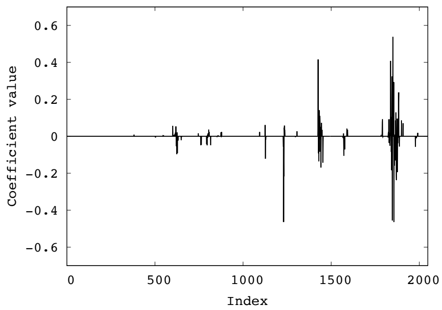

Fig. 4 Coefficients of DH (N=2048)

Table I. L1/L0-norms.

	DCT	Haar	DH
L1-norm	24.7	31.4	3.3
L0-norm	1024.0	1002.0	55.5

4. Conclusion

To obtain sparser representation of medical ultrasound signal, the overcomplete dictionary using DCT and Haar bases was investigated. The numerical simulation was conducted and the result shows that sparser L1 norm was obtained using the OD. In future work, a set of signals for a whole image will be applied simultaneously.

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