Improvement of Estimation Accuracy of Liver Fibrosis Parameters Based on Multi-Rayleigh Model Considering Fluctuation of Statistical Moment of Ultrasound Echo Envelope

超音波エコー振幅統計量の統計ゆらぎを考慮した
肝線維化パラメータ推定の高精度化

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1. Introduction

An analysis of amplitude distribution property of ultrasound echo envelope is usefull for realizing tissue characteristics. In previous studies, we proposed a multi-Rayleigh (MRA) model which expresses the probabillity distribution of echo envelope obtained from fibrotic liver[1]. Using the MRA model, quantitative liver fibrosis parameters can be estimated[1]. In the present study, we examined optimal input parameters for accurate estimation of liver fibrosis parameters based on the MRA model.

2. Multi-Rayleigh model for fibrotic liver

A probability density function (PDF) of ultrasound echo envelope from homogeneous tissue such as normal liver tissue can be approximated by a Rayleigh (RA) distribution as

$$p_{\mathrm{RA}}(x) = \frac{2x}{\sigma_{\mathrm{RA}}^2} \exp\left(-\frac{x^2}{\sigma_{\mathrm{RA}}^2}\right), \qquad (1)$$

where x is echo envelope amplitude and σ_{RA} is a scale parameter. On the other hand, the PDF of echo envelope from fibrotic liver tissue deviates from the RA distribution because of formation of fibrotic tissue. We proposed the combination model of plural RA distributions as a MRA model to express the PDF of echo envelope from fibrotic liver tissues[1]. The MRA model with two components is given as

$$p_{\mathrm{MRA}}(x) = \alpha_{\mathrm{L}} \cdot p_{\mathrm{LRA}}(x) + \alpha_{\mathrm{H}} \cdot p_{\mathrm{HRA}}(x), \quad (2)$$

where $p_{\mathrm{LRA}}(x)$ and $p_{\mathrm{HRA}}(x)$ are RA distributions for normal and fibrotic tissues, respectively. α_{L} and α_{H} are mixture rates of $p_{\mathrm{LRA}}(x)$ and $p_{\mathrm{HRA}}(x)$. The parameter $R_{\mathrm{m}} = \alpha_{\mathrm{H}}$ is a fiber mixture rate and $R_{\mathrm{v}} = \sigma_{\mathrm{HRA}}^2/\sigma_{\mathrm{LRA}}^2$ is a variance ratio of brightness between normal and fibrotic tissues.

3. Quantification of statistical fluctuation of moment of ultrasound echo envelope

MRA model parameters are estimated using statistical moments of echo envelope[2]. The n-th order of standardized moment is given as

$$M_n(x) = E\left[\frac{(x - \mu)^n}{\sigma^n}\right], \qquad (3)$$

where μ and σ are average and standard deviation of echo envelope x. The moment of measured ultrasound echo envelope is distributed following Gaussian distribution by statistical fluctuation[2] even if the liver tissue has the same structure and acoustic properties. This statistical fluctuation propagates to the MRA model parameters as estimation error so it is needed to examine optimal moments which reduce the estimation error of MRA model parameters.

In the present study, we examined the optimal combination of moments by quantifying a relationship between the statistical fluctuation of moment and the estimation error of MRA model parameters. Based on the law of error propagation, the standard deviation of estimation error e_{Ri} of R_i (i: m or v) in the MRA model is calculated as

$$\begin{aligned} e_{R_i}^2 &= \left(\frac{\partial R_i}{\partial M_{n1}}\right)^2 \cdot e_{M_{n1}}^2 + \left(\frac{\partial R_i}{\partial M_{n2}}\right)^2 \cdot e_{M_{n2}}^2 \\ &+ 2 \cdot \frac{\partial R_i}{\partial M_{n1}} \cdot \frac{\partial R_i}{\partial M_{n2}} \cdot \rho_{M_{n1}M_{n2}} \cdot e_{M_{n1}} \cdot e_{M_{n2}}, \end{aligned} \qquad (4)$$

where $e_{M_{n1}}$ and $e_{M_{n2}}$ are standard deviations of n_1-th and n_2-th orders of moments dispersed by statistical fluctuation and $\rho_{M_{n1}M_{n2}}$ is correlation coefficient between n_1-th and n_2-th orders of moments. It is difficult to calculate $\partial R_i/\partial M_{n_k}$ so that a transformation was used which is given as

$$\begin{aligned} &\begin{bmatrix} \dfrac{\partial R_{\mathrm{m}}}{\partial M_{n1}} & \dfrac{\partial R_{\mathrm{m}}}{\partial M_{n2}} \\ \dfrac{\partial R_{\mathrm{v}}}{\partial M_{n1}} & \dfrac{\partial R_{\mathrm{v}}}{\partial M_{n2}} \end{bmatrix} \\ &= \frac{1}{\dfrac{\partial M_{n1}}{\partial R_{\mathrm{m}}}\dfrac{\partial M_{n2}}{\partial R_{\mathrm{v}}} - \dfrac{\partial M_{n1}}{\partial R_{\mathrm{v}}}\dfrac{\partial M_{n2}}{\partial R_{\mathrm{m}}}} \begin{bmatrix} \dfrac{\partial M_{n2}}{\partial R_{\mathrm{v}}} & \dfrac{\partial M_{n1}}{\partial R_{\mathrm{v}}} \\ \dfrac{\partial M_{n2}}{\partial R_{\mathrm{m}}} & \dfrac{\partial M_{n1}}{\partial R_{\mathrm{m}}} \end{bmatrix}, \end{aligned} \qquad (5)$$

where the discrete differential value of $\partial M_{n_k}/\partial R_i$ can be numerically calculated. The standard deviation of moment, e_{M_n}, and correlation coefficient $\rho_{M_{n1}M_{n2}}$ were calculated by numerical simulation with generating random numbers following the MRA model. For given MRA model parameters ($R_{\mathrm{m}} = 0.2$, $R_{\mathrm{v}} = 4.0$), the standard
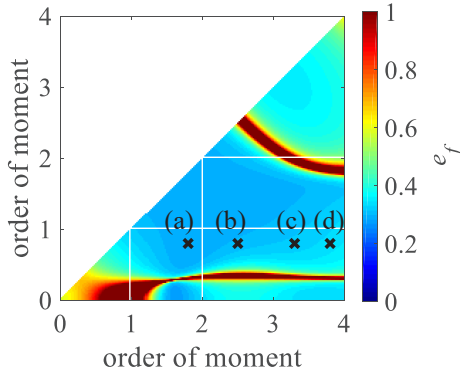
Fig. 1   Magnitude of statistical fluctuation for liver fibrosis parameters evaluated by e_f in Eq. (6). Values of e_f were (a) 0.28, (b) 0.29, (c) 0.31, and (d) 0.33.

deviations e_{R_m} and e_{R_v} of R_m and R_v were numerically calculated for each combination of moments using Eqs. (4) and (5). Then, a magnitude value e_f of statistical fluctuation in liver fibrosis parameters (R_m, R_v) was calculated as

$$e_f = \sqrt{\left(\frac{e_{R_\mathrm{m}}}{R_\mathrm{m}}\right)^2 + \left(\frac{e_{R_\mathrm{v}}}{R_\mathrm{v}}\right)^2}, \qquad (6)$$

where $R_\mathrm{m} = 0.2$ and $R_\mathrm{v} = 4.0$. **Figure 1** shows calculated results of e_f for each combination of moments. The effect of statistical fluctuation in liver fibrosis parameters were changed by combinations of moments used for estimating the MRA model.

4. Simulation and Result

A relationship between the magnitude e_f of statistical fluctuation in liver fibrosis parameters and estimation accuracy of liver fibrosis parameters was examined using simulated ultrasound B-mode images. The ultrasound B-mode image (**Fig. 2(b)**) was simulated from the tissue structural model in **Fig. 2(a)** using the simulation tool of Field II. In the tissue structural model, zonal fibrotic tissues with 2.0 mm of width were arranged in the normal tissues; therefore, the fiber mixture rate R_m was set to be 0.2. The difference of normal and fibrotic tissues was expressed by changing the reflection coefficient such that the fiber variance ratio R_m was set to be 4.0. **Figure 2(c)** shows the PDF of echo envelope in Fig. 2(b) and estimated MRA model with two components. For the simulated B-mode image, liver fibrosis parameters (R_m, R_v) were estimated using the several combinations of moment denoted by X marks in Fig. 1. Liver fibrosis parameters were estimated for 1000 types of B-mode images. The B-mode images have identical tissue structure and acoustic properties, and have only the difference of arrangement of scattered points. **Figure 3** shows the standard deviations of estimated liver fibrosis parameters. The standard deviations of liver fibrosis parameters became small by using the combination of moments which decreases the magnitude e_f of statistical fluctuation as the input
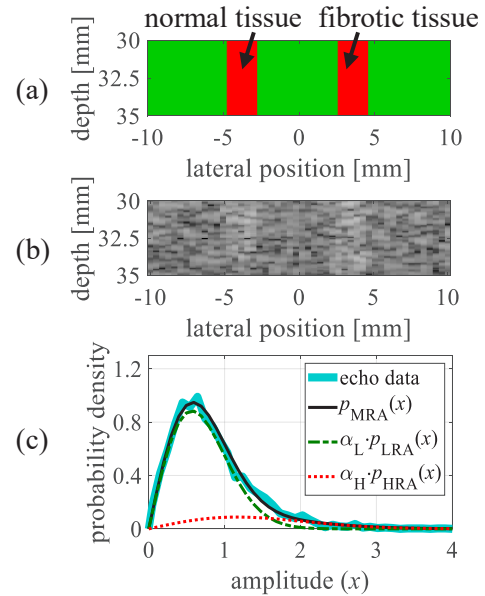

Fig. 2   (a) Tissue structural model, (b) simulated B-mode image, and (c) estimated MRA model.
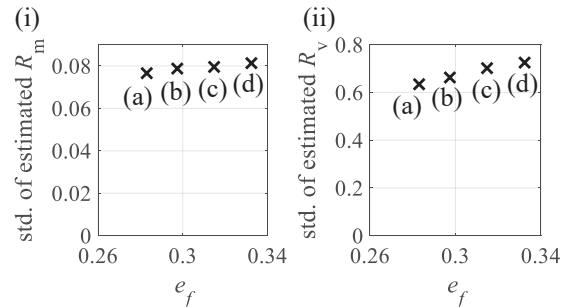

Fig. 3   Standard deviations of (i) estimated R_m and (ii) estimated R_v using each combinations of moments, (a) - (d), in Fig. 1.

parameters in the MRA model estimation.

5. Conclusion

In the present study, we examined the optimal combination of moments used in the MRA model estimation for accurate estimation of liver fibrosis parameters. The effect of the statistical fluctuation of moments in the MRA model estimation was quantified based on the law of error propagation. It was suggested that the estimation accuracy of liver fibrosis parameters can be improved by using the combination of moment which decreases the effect of statistical fluctuation in the MRA model estimation. An establishment of the accurate estimation method for liver fibrosis parameters with minimizing the effect of statistical fluctuation is a future task.

References
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