3J2-1

Resonance theory of elastic waves scattered from an elastic cylinder with a spring interface

ばね界面を有する弾性体円柱における弾性波の共鳴散乱理論

Kazusa Yamaguchi[1‡], Naoki Matsuda[2], Masaaki Nishikawa[2], and Masaki Hojo[2]
([1]Grad. School Eng., Kyoto Univ.; [2]Kyoto Univ.)
山口知紗 [1‡]，松田直樹 [2]，西川雅章 [2] ，北條正樹 [2] （[1] 京大院　工，[2] 京大　工）

1. Introduction

The resonance scattering of acoustic or elastic waves from an elastic cylinder has been investigated theoretically and experimentally.[1-3] Resonance scattering is due to the eigen vibration of the cylinder when the frequency of the incident wave coincides with the eigen frequency of the cylinder. This decreases the amplitude of the backscattering wave.

In most studies on elastic wave resonance scattering, the cylinder is assumed to be perfectly connected to the matrix. However, actual contact or adhesive interfaces should be considered imperfect boundaries. The spring interface model is a widely used numerical model for this purpose. Elastic scattering wave for the model has been studied,[4] but resonance scattering is not mentioned in detail.

In this paper, we propose a resonance theory of elastic wave scattering for the spring interface model and then demonstrate its feasibility through a numerical example.

2. Theory

We consider a system in which an elastic cylinder with radius a is connected to an elastic matrix by springs. In **Fig. 1**, K_r and K_θ denote the interfacial stiffnesses in the radial and angular directions, respectively. We distinguish the physical quantities of the incident, scattered, and refracted wave fields by superscripts: inc, sca, and ref, respectively.

We assume that an infinite-plane longitudinal wave with an angular frequency ω is incident on the cylinder. The amplitude of the far-field scattering wave is known as form function. The longitudinal scattering form function f^{pp} and the transverse scattering form function f^{ps} can be expressed as a sum of the partial wave amplitudes:

$$f^{\mathrm{pp}}(\theta) = \sqrt{\frac{2}{\pi i k_{\mathrm{L}} a}} \sum_{n=-\infty}^{\infty} A_n \cos(n\theta), \quad (1)$$

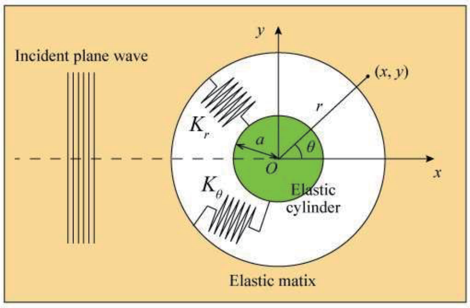

Fig. 1　Geometry of an elastic cylinder with a spring interface. The gap between the cylinder and the matrix is ignored.

$$f^{\mathrm{ps}}(\theta) = \sqrt{\frac{2}{\pi i k_{\mathrm{T}} a}} \sum_{n=-\infty}^{\infty} B_n \sin(n\theta). \quad (2)$$

Here and in the following, the $\exp(-i\omega t)$ time dependence is suppressed. In Eqs. (1) and (2), $k_{\mathrm{L}} = \omega / c_{\mathrm{L}}$ and $k_{\mathrm{T}} = \omega / c_{\mathrm{T}}$ are wavenumbers, where c_{L} and c_{T} denote the longitudinal and transverse velocities of the matrix, respectively. The integer n represents the normal mode of the partial wave. The expansion coefficients A_n and B_n can be determined from the boundary condition for the spring interface at $r = a$:

$$\sigma_{rr}^{\mathrm{inc}} + \sigma_{rr}^{\mathrm{sca}} = \sigma_{rr}^{\mathrm{ref}}, \quad (3)$$

$$\sigma_{r\theta}^{\mathrm{inc}} + \sigma_{r\theta}^{\mathrm{sca}} = \sigma_{r\theta}^{\mathrm{ref}}, \quad (4)$$

$$\sigma_{rr}^{\mathrm{inc}} + \sigma_{rr}^{\mathrm{sca}} = K_r\left(u_r^{\mathrm{inc}} + u_r^{\mathrm{sca}} - u_r^{\mathrm{ref}}\right), \quad (5)$$

$$\sigma_{r\theta}^{\mathrm{inc}} + \sigma_{r\theta}^{\mathrm{sca}} = K_\theta\left(u_\theta^{\mathrm{inc}} + u_\theta^{\mathrm{sca}} - u_\theta^{\mathrm{ref}}\right). \quad (6)$$

When $K_r, K_\theta \to \infty$, the boundary condition is the same as the case for the perfect bond.[2] According to Rhee and Park,[3] the expansion coefficients A_n and B_n can be expanded as follows:

$$A_n = A_n^{(*)} + \frac{1}{2} S_n^{(*)\mathrm{res,pp}} + A_n^{(*)} S_n^{(*)\mathrm{res,pp}}, \quad (7)$$

$$B_n = B_n^{(*)} + B_n^{(*)} S_n^{(*)\mathrm{res,ps}}, \quad (8)$$

where superscript (*) indicates that the quantity is related to rigid (r) or soft (s) cylinders, and $S_n^{(*)\mathrm{res,pp}}$ and $S_n^{(*)\mathrm{res,ps}}$ are resonance scattering functions, expressed as,

Table I Physical properties of materials

Material	Mass density $\rho(\mathrm{kg/m^3})$	Longitudinal wave velocity $c_{\mathrm{L}}(\mathrm{m/s})$	Transverse wave velocity $c_{\mathrm{T}}(\mathrm{m/s})$
Aluminum (Matrix)	2700	6420	3132
Steel (Cylinder)	7840	5908	3205

$$S_n^{(*)\mathrm{res,pp}} = 2\frac{A_n - A_n^{(*)}}{1+2A_n^{(*)}}, \quad (9)$$

$$S_n^{(*)\mathrm{res,ps}} = \frac{B_n - B_n^{(*)}}{B_n^{(*)}}. \quad (10)$$

Applying Eqs. (7) and (8) to Eqs. (1) and (2), respectively, we can express the form functions purely in terms of the resonance scattering functions:

$$f^{(*)\mathrm{res,pp}}(\theta) = \sum_{n=-\infty}^{\infty} f_n^{(*)\mathrm{res,pp}}(\theta), \quad (11)$$

$$f^{(*)\mathrm{res,ps}}(\theta) = \sum_{n=-\infty}^{\infty} f_n^{(*)\mathrm{res,ps}}(\theta), \quad (12)$$

where the partial wave form functions $f_n^{(*)\mathrm{res,pp}}$ and $f_n^{(*)\mathrm{res,ps}}$ are given by,

$$f_n^{(*)\mathrm{res,pp}}(\theta) = \frac{1}{\sqrt{2\pi i k_{\mathrm{L}} a}} S_n^{(*)\mathrm{res,pp}} \cos(n\theta), \quad (13)$$

$$f_n^{(*)\mathrm{res,ps}}(\theta) = \frac{1}{\sqrt{2\pi i k_{\mathrm{T}} a}} S_n^{(*)\mathrm{res,ps}} \sin(n\theta). \quad (14)$$

According to the resonance theory, each partial wave is considered to affect the total scattering field at the corresponding resonance frequency. We demonstrate this phenomenon through a numerical example in the next section.

3. Numerical results and discussion

We present the numerical result for the longitudinal scattering case. **Table I** lists the material parameters used. The nondimensional interfacial stiffnesses are taken as $K_r a/\mu = 20$ and $K_\theta a/\mu = 20 \times 0.3$, where μ is the Lamé constant of the matrix. The maximum order of the normal mode n is equal to $n = k_{\mathrm{L}} a_{\max} + 5$ instead of ∞, where $k_{\mathrm{L}} a_{\max}$ is the maximum value of the nondimensional frequency $k_{\mathrm{L}} a$ in the numerical analysis. Here, we consider the rigid case (r) in Eq. (13), because the cylinder is considered to be made of steel and is known to behave as a rigid body except at resonance frequencies.

Figure 2(a) shows the amplitude of the form function $|f^{\mathrm{pp}}(\pi)|$ calculated using Eq. (1). Figures 2(b)–(e) show the amplitude of the partial wave form function $\left|f_n^{(\mathrm{r})\mathrm{res,pp}}(\pi)\right|$ (for $n = 0, \dots, 3$) obtained using Eq. (13). The horizontal axes in Fig. 2 represent the nondimensional frequency $k_{\mathrm{L}} a$. Several peaks can be observed in Figs. 2(b)–(e), corresponding to each local minimum in Fig. 2(a). The same trend can be seen in the case of transverse scattering. This indicates that the total backscattering amplitude decreases because of the partial wave at the resonance frequency. This result is consistent with those of previous research[1,3] and shows that the resonance theory can be extended to cases with a spring boundary condition.

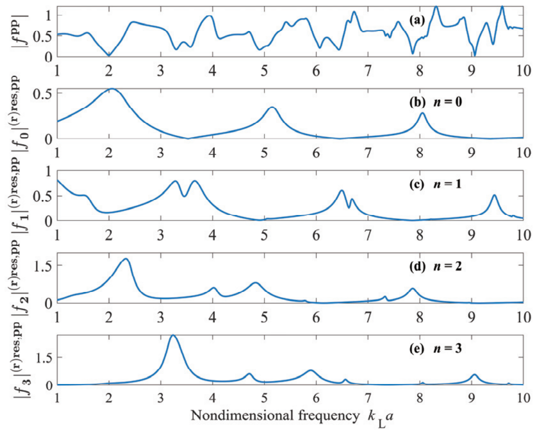

Fig. 2 (a) Form function for longitudinal scattering and (b)–(e) partial wave form functions for the first fourth wave modes ($n = 0, \dots, 3$) for backscattering.

4. Conclusions

In this paper, we propose a resonance theory for the backscattering by an elastic cylinder with a spring interface. The numerical results of the form function and the partial wave form functions for backscattering are given. We prove that the resonance theory can be extended correctly to the case with a spring boundary condition.

References

1. L. Flax, L. R. Dragonette and H. Überall: J. Acoust. Soc. Am. **63** (1978) 723.
2. Y. Fan, A. N. Sinclair and F. Honarvar: J. Acoust. Soc. Am. **106** (1999) 1229.
3. H. Rhee and Y. Park: JSME Int. J., Ser. C **47** (2004) 297.
4. W. Huang, S. Brisuda and S. I. Rokhlin: J. Acoust. Soc. Am. **97** (1995) 807.