

## Optimal Subcarrier Design for OFDM in Channels with Nonlinear Distortion

非線形歪みを有する通信路における OFDM の最適なサブキャリアの設計

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### 1. Introduce

Recently, the development of wireless portable devices has increased the demand for efficient and reliable wireless communication scheme. Orthogonal frequency division multiplexing (OFDM) is a key communication scheme that satisfies such demands: OFDM achieves excellent frequency usage, while preserves small calculation cost for equalization. However, the peak to average power ratio (PAPR) of OFDM signal is higher than other modulation signals. Then, nonlinear distortion caused by amplifiers in the transmitter seriously affect communication quality. As previous studies, selected mapping (SLM)<sup>[1]</sup> and partial transmit sequence (PTS)<sup>[2]</sup> methods exist for reducing the PAPR of the OFDM signal. In this paper, we propose an alternative to maintain the communication quality even in the channel with nonlinear distortion by designing OFDM subcarrier allocation appropriately.

### 2. Proposed OFDM subcarrier design

**Figure 1** shows the block diagram of the proposed OFDM communication system. The channel with nonlinear distortion is composed of two amplifiers, a loudspeaker, acoustic delay line, and a microphone. **Figure 2** shows the structure of OFDM signal and proposed subcarrier allocation. Let  $T$  denote the signal duration and  $T_g$  the guard interval (GI) [Fig. 2(a)]. Then, subcarriers are located at frequency

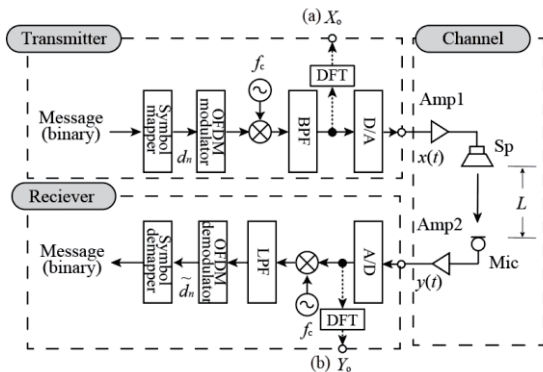


Fig. 1 Block diagram of proposed OFDM communication system.

$$f_k = f_c + \frac{k}{T} \quad (k = -\frac{K}{2}, \dots, -1, 0, 1, \dots, \frac{K}{2} - 1) \quad (1)$$

$$\mathbf{f}_{\text{sub}} := (f_{-K/2}, \dots, f_{-1}, f_0, f_1, \dots, f_{K/2-1}) \quad (2)$$

where  $f_c$  is the carrier frequency,  $K$  is the total number of subcarriers and  $\mathbf{f}_{\text{sub}}$  is a vector of length  $K$  whose component is subcarrier frequency  $f_k$ . We define  $\mathbf{f}_{\text{act}}$  and  $\mathbf{f}_{\text{null}}$  as active and null subcarrier frequency sets, respectively. Then,  $\mathbf{f}_{\text{act}}$  and  $\mathbf{f}_{\text{null}}$  are determined as

$$\mathbf{f}_{\text{act}} = (f_{a(1)}, f_{a(2)}, \dots, f_{a(N)}) \in \mathbf{f}_{\text{sub}} \quad (3)$$

$$\mathbf{f}_{\text{null}} = \mathbf{f}_{\text{sub}} \cap \overline{\mathbf{f}_{\text{act}}} \quad (4)$$

where  $f_{a(n)}$  ( $n=1, \dots, N$ ) are active subcarrier frequency,  $N$  is the total number of active subcarriers, and the total number of null subcarriers becomes  $(K-N)$ .

Then, let  $d_n$  denote a complex symbol converted from binary messages. OFDM baseband signal  $x(t)$  is given as

$$x(t) = \sum_{n=1}^N d_n \exp\left(j2\pi \frac{f_{a(n)}}{T} t\right), \quad t \in [0, T]. \quad (5)$$

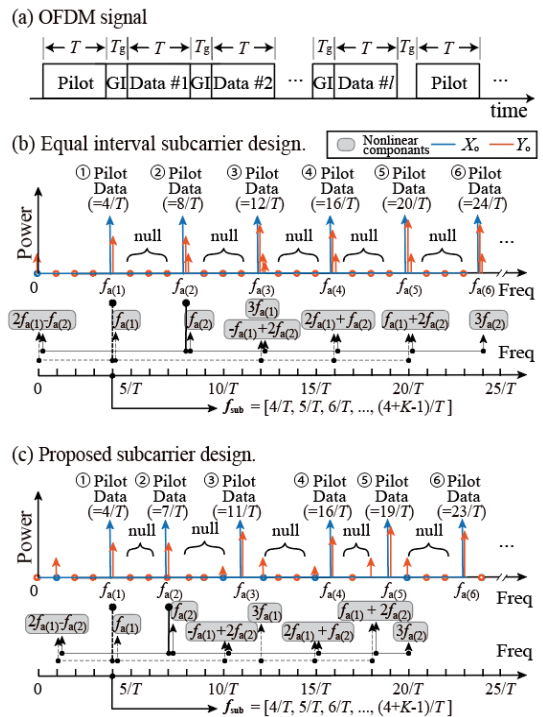


Fig. 2 Structure of OFDM signal and proposed subcarrier allocation design.

Then, Fig. 2(b), (c) shows  $X_o$  and  $Y_o$  which are the discrete Fourier transform (DFT) of OFDM signal and receive signal [Fig. 1(a), (b)]. In this figure,  $f_{\text{sub}}$  is set as  $[4/T, 5/T, \dots, (4+K-1)/T]$ . Generally, the nonlinear distortion caused by the amplifier with symmetrical clip characteristics produces odd-order nonlinear components. Thus, to simplify, we focus only on the 3rd order nonlinear components of  $f_{a(1)}$  and  $f_{a(2)}$ ;  $f_{a(1)}, f_{a(2)}, 2f_{a(1)} \pm f_{a(2)}, f_{a(1)} \pm 2f_{a(2)}, 3f_{a(1)}, 3f_{a(2)}$ . In Fig. 2(b) that is the subcarrier design of equal intervals ( $f_{\text{act}} = [4/T, 8/T, 12/T, 16/T, 20/T, 24/T, \dots]$ ), all nonlinear components do overlap with active subcarriers. On the other hand, In Fig.2 (c) that is proposed subcarrier design ( $f_{\text{act}} = [4/T, 7/T, 11/T, 16/T, 19/T, 23/T, \dots]$ ), all nonlinear components, except for  $f_{a(1)}$  and  $f_{a(2)}$  do not overlap with active subcarriers. Therefore, in this paper, we design active subcarriers  $f_{a(n)}$  ( $n=1, \dots, N$ ) according to following policy.

- (A1) The greatest common divisor of any  $N$  numbers  $f_{a(n)}$  is  $1/T$ .
- (A2) The difference between  $f_{a(n)}$  and  $f_{a(n+1)}$  is not constant.

### 3. Experiment

We evaluate the performance of the proposed subcarrier design in the experiment. The communication system consists of digital to analog converter (NI USB-6212), analog to digital converter (NI USB-6212), two operational amplifiers (Amp1: LM386, Amp2: JRC4580), a loudspeaker (FOSTEX P800K) in the speaker box (P800E) and a microphone (C9767). Table 1 shows experimental parameters. Since the OFDM signal duration  $T$  is 1 (s),  $f_{\text{sub}}$  is determined as  $[2000 \ 2001 \ \dots \ 9999]$  (Hz) from Equation (1) and (2). Then, We set reference subcarriers and three types of proposed active subcarriers  $f_{a(n)}$  ( $n=1, \dots, 800$ ) as

- (Ref.)  $f_{a(n)} = 1990 + 10n$  (Hz).
- (Prod.1)  $f_{a(n)} = 1991 + 10n$  (Hz). (A1)
- (Prod.2) Prime numbers in  $f_{\text{sub}}$  (Hz). (A1) (A2)  
 $[f_{a(1)}=2003, f_{a(2)}=2011, \dots, f_{a(800)}=9991]$ .
- (Prod.3) Random numbers in  $f_{\text{sub}}$  (Hz). (A1) (A2)  
 $[f_{a(1)}=2002, f_{a(2)}=2012, \dots, f_{a(800)}=9995]$ .

We measure each BERs by increasing the input level for four types OFDM signal. Figure 3 shows the experimental result. In this figure, relative input power (RIP) (dB) is calculated based on the

Table. 1 Experimental parameter.

Parameters	Value
Distance between sp and mic (m)	$L$ 0.5
Number of data packet	$l$ 20
Each packet Signal length (s)	$T$ 1
Guard interval (s)	$T_g$ 0.2
Transmission Quantity (bps)	1600
Modulation	QPSK
Sampling frequency (kHz)	96
Carrier frequency (kHz)	$f_c$ 6
Signal bandwidth (kHz)	2-10
Number of subcarriers	$K$ 8000
Number of pilot (data) subcarriers	$N$ 800
Number of null subcarriers	$K-N$ 7200

power when the reference method becomes  $\text{BER} > 10^{-5}$ . Fig. 3 indicates that the proposed subcarrier design (red, purple and green colored line) decided by the policy (A1) and (A2) reduces the BER in the region where the nonlinear distortion occurs ( $\text{RIP} > 0$ ). Furthermore, random subcarrier allocation (purple colored line) according to both (A1) and (A2) achieves the best BER characteristic.

Now, we focus on PAPR given by

$$\text{PAPR} = 10 \log_{10} \left( \frac{\max_{t \in [0, T]} (x(t)^2)}{\text{mean}_{t \in [0, T]} (x(t)^2)} \right) \quad (6)$$

where  $\max(x)$  and  $\text{mean}(x)$  return the maximum and average value of the argument  $x$ , respectively. Figure 4 shows the probability distribution of PAPR  $\text{Pr}(\text{PAPR})$  for each OFDM signal. This figure indicates that the random allocation OFDM (painted green) that achieves smallest BER has the highest PAPR. This result indicates that the way to maintain the communication quality in the channel with nonlinear distortion is to design the OFDM active subcarriers appropriately, rather than simply reducing the PAPR of the OFDM signal.

### 4. Conclusion

We proposed active subcarrier design policy (A1) and (A2) and confirm the feasibility through the BER result. From the result, we consider the truly optimal OFDM subcarriers are able to be designed according to (A1) and (A2) strictly.

### References

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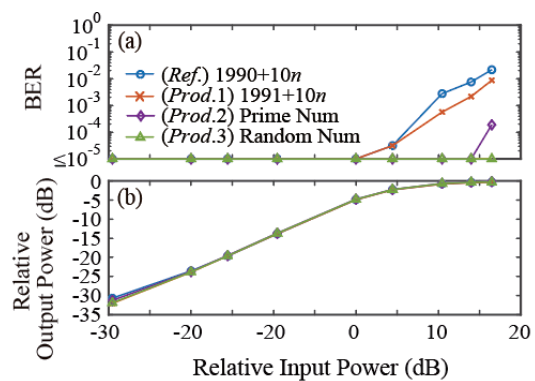


Fig. 3 The BERs averaged over each packet.

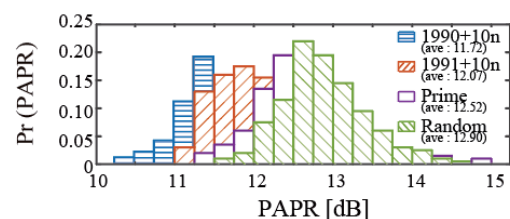


Fig. 4 Probability density of PAPR.