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Liquid Surface Deformation Caused by Acoustic Radiation Force of Focused Ultrasound
集束超音波の音響放射力による液面形状変化の検討

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1. Introduction

The impinging of ultrasounds into a liquid-gas interface deforms the surface shape. The balance between acoustic radiation pressure (ARP), gravity, and surface tension determines this surface shape. Based on this relationship, we have proposed the simultaneous measurements of ARP and sound pressure distributions and ultrasound power[1,2]. Although the theoretically predicted displacement is proportional to the square of the sound pressure, however, the experimentally obtained displacement is proportional to the third power.

The purpose of this study is to consider the theory for predicting surface displacement induced by the ARP of focused ultrasound. In particular, small deformations was assumed in previous studies[1,2], however, large deformation of a surface is taken into account in this study.

2. Theory

An ultrasound transducer in liquid (density ρ_1 and sound speed c_1) emits ultrasounds into an interface between liquid and gas (density ρ_2 and sound speed c_2) as shown in **Fig. 1**. In a steady-state, the surface displacement (height) h from the rest surface is determined by ARP P_{ARP} and the surface tension, and this is described by

$$-(\rho_1 - \rho_2)gh + P_{\mathrm{ARP}} + \sigma\kappa = 0, \quad (1)$$

where g is the gravitational acceleration, σ is the surface tension. If the sound field is axisymmetry around the acoustic axis z, the curvature κ is given by

$$\kappa = \frac{\dfrac{d^2h}{dr^2}}{\left[1+\left(\dfrac{dh}{dr}\right)^2\right]^{3/2}} + \frac{1}{r}\frac{\dfrac{dh}{dr}}{\sqrt{1+\left(\dfrac{dh}{dr}\right)^2}}, \quad (2)$$

where r is the radial distance from the axis.

The ARP perpendicular to the surface is given by

$$P_{\mathrm{ARP}} = (\boldsymbol{P}_{1\mathrm{ARP}} - \boldsymbol{P}_{2\mathrm{ARP}}) \cdot \boldsymbol{n}, \quad (3)$$

$$\boldsymbol{P}_{i\mathrm{ARP}} = -\langle \mathcal{L}_i \rangle \boldsymbol{n} + \rho_1 \langle \boldsymbol{u}_i (\boldsymbol{u}_i \cdot \boldsymbol{n}) \rangle, \quad (4)$$

where the subscript i =1 and 2 indicates mediums of the liquid and gas, respectively, $\boldsymbol{n}$ means the normal unit vector from the liquid to gas, $\mathcal{L}_i$ is the Lagrangian density

$$\mathcal{L}_i = \frac{1}{2}\rho_i \boldsymbol{u}_i \cdot \boldsymbol{u}_i - \frac{p_i^2}{2\rho_i c_i^2}, \quad (5)$$

and p_i and $\boldsymbol{u}_i$ are sound pressure and particle velocity, respectively. The bracket $\langle\cdot\rangle$ means time averaging operator.

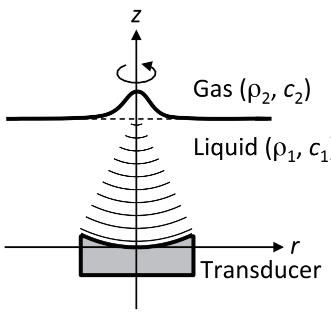

Fig. 1 Theoretical model.

In order to predict the surface displacement induced by ultrasound, first, we calculate sound fields including reflections and transmissions. However, multiple reflections are not considered.

Here, we consider a concave transducer with the aperture radius a and the curvature of radius of emitting surface D. The velocity potential $\phi(r,z,t) = q(r,z)\exp(j\omega t - kz)$ of incident ultrasound at an angular frequency of $\omega = 2\pi f$ of sound pressure P_0 on the surface is obtained as

$$q(r,z) = \frac{P_0}{\rho_1 c_1 z}\exp\left(-j\frac{kr^2}{2D}\right) \times \int_0^a \exp\left[j\frac{kr'^2}{2}\left(\frac{1}{D}-\frac{1}{z}\right)\right] J_0\left(\frac{krr'}{z}\right) r'dr', \quad (6)$$

under the parabolic approximation, where $k_1 = \omega/c_1$ is the wavenumber in the liquid and $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind[3]. p_1, $\boldsymbol{u}_1$, p_2, and $\boldsymbol{u}_2$ on the boundary are obtained from the incident ultrasound by considering the reflection and transmission coefficients at the interface.

Then, using the obtained sound fields, we predict the displacement of surface using Eqs. (1)–(5). Since Eq. (1) substituting. Eqs. (2)–(5) becomes

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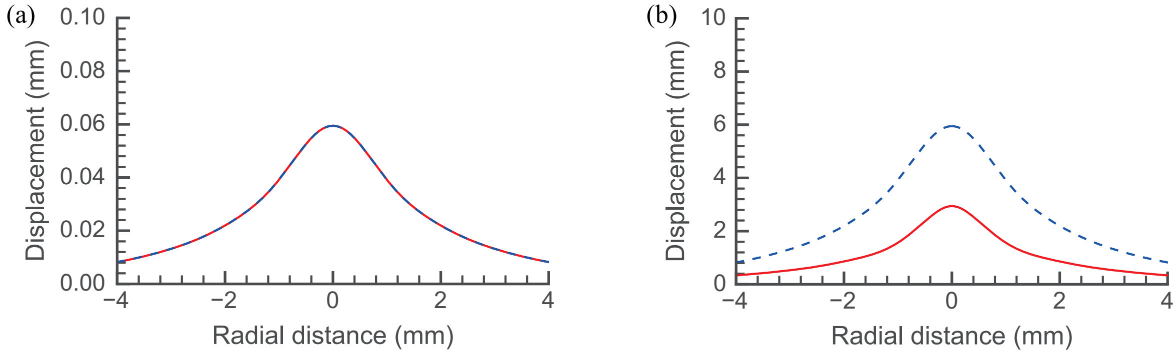

Fig. 2 Theoretically predicted surface displacement. Solid and dashed lines indicate the predictions of large- and small-deformation theories, respectively. (a) Sound pressures of 100 kPa and (b) of 1000 kPa.

a nonlinear partial differential equation, it is hard to solve directly. Therefore, an iteration calculation is performed. In the first step, displacement $\tilde{h}$ is calculated on the assumption $dh/dr = 0$ in the denominators on the right-hand side of Eq. (2). In the second step, displacement h is calculated using the denominators of Eq. (2) as constants using the values obtained in the first step.

3. Results and discussion

As numerical examples, we calculated surface displacements of water and air interface induced by ultrasound at f=2.8 MHz radiated from a concave transducer of D=7.6 cm and a=1.7 cm. The water is filled up to the depth at which the sound pressure is maximized.

Figure 2 shows surface displacement predicted by the present theory at different sound pressure. As the reference, small-deformation theory on the assumption with $dh/dr \approx 0$ is plotted with together. The result indicates that although both the predictions agree well at 100 kPa, however, the prediction for the large-deformation theory is lower than that for the small-deformation theory at 1000 kPa.

The relationship between the maximum displacement of the water surface and the sound pressure is plotted in **Fig. 3**. The small-deformation theory shows that the displacement is proportional to the square of the sound pressure, while the large-deformation theory shows that the increase rate decreases as the sound pressure increases.

The decrease of the displacement may be due to the fact that the surface gradient reduces the effective acoustic radiation force. However, the displacement of the third power characteristic obtained in experiments[1,2] was not observed in the present study. Phenomena that are not considered in this theory, such as acoustic streaming and multiple reflections in a water mound, maybe the cause of the difference between experiment and theory.

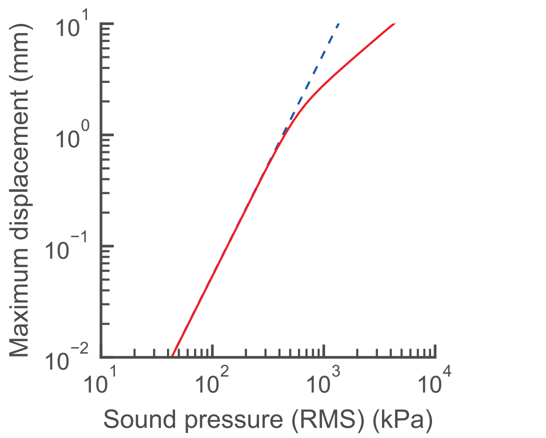

Fig. 3 Relationship between the maximum displacement and sound pressure. Solid and dashed lines indicate the predictions of large- and small-deformation theories, respectively.

4. Conclusion

In this study, to predict the surface deformation induced by ARP, we considered a theory considering large-deformation of the surface. Numerical examples showed that the effect of large-deformation decrease the displacement, however, the increase rate of displacement did not agree with experiments[1,2].

For future work, it is necessary to consider acoustic streaming and multiple reflections ignored in the present model and accurate displacement measurement methods.

Acknowledgment

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