

## Characteristic analysis on linear regression beamformer 線形回帰ビームフォーマの特性解析

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### 1. Introduction

Ultrasound beamforming is a very important process to form an ultrasonic image. Beamforming is required to enhance an ultrasonic echo from a point of interest (receiving focal point) and suppresses out-of-focus echoes. Currently, delay-and-sum (DAS) beamforming is widely used in clinical ultrasound scanners. However, the ability of the DAS beamformer to suppress out-of-focus echoes is limited and various adaptive beamformers have been introduced in medical ultrasound imaging.

To evaluate coherence among ultrasonic echo signals received by individual transducer elements, the coherence factor (CF) was introduced [1,2]. The CF evaluates the ratio of the energy of coherent components to the total received energy. Therefore, the CF obtained from echoes containing strong out-of-focus echoes compared with an echo from a point at the focus becomes low and such an echo is suppressed by weighting the beamformed signal with the CF. More recently, other metrics, such as short-lag spatial coherence (SLSC) [3,4], have been introduced to evaluate coherence among element echo signals.

Minimum variance (MV) beamforming was also introduced in medical ultrasound imaging to improve image quality [5-7]. A MV beamformer determines the weights to element echo signals so that the power of the beamformer output is minimized, i.e., noise, including out-of-focus echoes, is minimized. The difference between the CF and MV beamformer is that the CF suppresses an echo signal with strong out-of-focus echoes even when it contains an echo from a point at the focus while the MV beamformer try to suppress out-of-focus echoes by optimizing the beamforming weights. Although the MV beamformer was shown to improve ultrasound image quality significantly, its high computational cost prevents it from being used in clinical situations.

We developed a new beamformer, namely, linear regression (LR) beamformer, to improve the ability to suppress out-of-focus echoes. In the present study, the fundamental characteristic of the LR beamformer was investigated.

### 2. Linear regression (LR) beamforming

As illustrated in Fig. 1, the arrival time of an echo from a spatial point is different element by element owing to the difference in the propagation distance between the spatial point to each element.

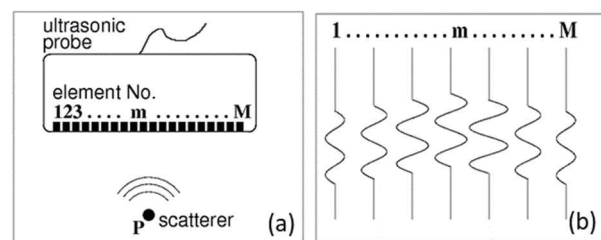


Fig. 1: Illustration of reception of ultrasonic echo with transducer array. (a) Array transducer and scatterer. (b) Echo signals received by individual transducer elements.

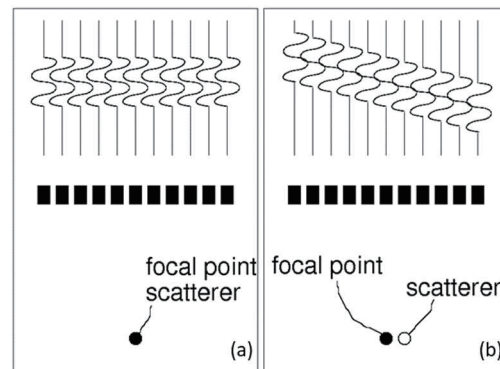


Fig. 2: Received echo signals delayed on the basis of DAS beamforming. Focal point and scatterer are located at (a) the same position and (b) different positions.

Element echo signals are aligned as shown in Fig. 2(a) when the focal point is located at the position of a scatterer. In this case, the echo signal is enhanced by summing the element echo signals across the array. On the other hand, element echo signals are not aligned when the focal point is not located at the position of the scatterer. In such a case, the echo signal is suppressed by summing the element echo signals. The DAS beamformer enhances an echo from a focal point and suppresses out-of-focus echoes based on such a principle.

Let us denote ultrasound echo signal received by the  $m$ -th transducer element by  $s_m$ , where  $s_m$

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is already delayed on the basis of conventional DAS beamforming.

After the delay compensation, an echo from a receiving focal point contained in  $\mathbf{S}$  becomes a direct current (DC) component across the receiving aperture. Therefore, in the conventional DAS beamforming, the output of the beamformer  $y_{\text{DAS}}$ , which corresponds to an echo from a focal point  $y$ , is obtained by averaging the element echo signals  $s_m$  after delay compensation as follows:

$$\hat{y}_{\text{DAS}} = \frac{1}{M} \sum_{i=0}^{M-1} s_i, \quad (1)$$

In the present study, the accumulated element signals  $u_m$  ( $m = 0, 1, \dots, M-1$ ) are defined as follows:

$$u_m = \sum_{i=0}^{m-1} s_i, \quad (2)$$

where  $u_0 = 0$ .

As described above, an echo  $y$  from a focal point becomes a DC component in element echo signals  $s_m$  after delay compensation. Therefore, the accumulated element echo signal  $u_m$  is modeled as follows:

$$\hat{u}_m = y \cdot m + n, \quad (3)$$

where  $n$  is a bias induced by additive noise. Let us define the mean squared difference  $\alpha$  between the measured accumulated element signal  $u_m$  and the model  $\hat{u}_m$  as:

$$\alpha = \sum_{m=0}^{M-1} \{u_m - (y \cdot m + n)\}^2. \quad (4)$$

The least-square estimates  $\hat{y}$  and  $\hat{n}$  of the echo from the focal point and the bias due to noise are obtained by setting the partial derivatives of  $\alpha$  with respect to  $y$  and  $n$  to zero as follows:

$$(\hat{y}, \hat{n}) = \arg \min_{y,n} \alpha. \quad (5)$$

The least-square difference  $\alpha_{\min}$  is obtained by substituting  $\hat{y}$  and  $\hat{n}$  into Eq. (4). In the present study, a weighting factor  $w$ , which emphasize an echo from a focal point, was defined as follows:

$$w = \frac{|\hat{y}|^2}{\alpha_{\min}}. \quad (6)$$

The residual difference  $\alpha_{\min}$  is very small when out-of-focus echoes are hardly included in received echo signals and the weight factor  $w$  becomes extremely large (unstable). Therefore, in the present study, stabilization parameter  $\gamma$  ( $\geq 0$ ) was introduced as follows:

$$w = \frac{|\hat{y}|^2}{\alpha_{\min} + \gamma |\hat{y}|^2}. \quad (7)$$

The final output  $y_{\text{LR}}$  of the proposed LR beamformer is obtained as follows:

$$\hat{y}_{\text{LS}} = w \cdot \hat{y}. \quad (8)$$

### 3. Characteristic analysis of LR beamformer

The lateral directivities of the DAS and LR beamformers were investigated by simulation experiments. In this simulation, an ultrasonic echo is assumed to be coming from different directions under the far field approximation. The ultrasonic center frequency was set at 7.5 MHz. Figure 3 shows the lateral directivities of the DAS and LR beamformers. As can be seen in Fig. 3, the LR beamformer suppresses out-of-focus echoes at the expense of the lateral resolution.

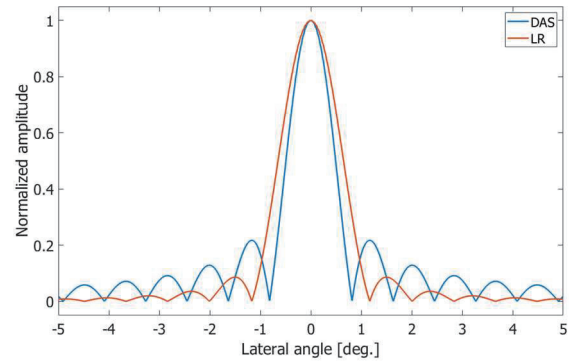


Fig. 3: Lateral directivities obtained by DAS and LR beamforming.

### 6. Conclusion

In the present study, the characteristic of the LR beamformer developed by our group was analyzed by simulation experiments. The simulation experimental result show that the LR beamformer suppresses out-of-focus echoes at the expense of the lateral resolution.

### References

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