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Further Correction of Measurement and Inference Errors in Piezoelectric Equivalent Inductance Components
圧電等価インダクタンスの測定誤差・推定誤差の更なる補正

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1. Introduction

In the equivalent circuit of piezoelectric transducer shown in **Fig. 1**, we have already shown the precise estimation method for $C/C_0 = \Sigma C_n/C_0$ and L_n. However, some "inference" process is required for this estimation. In this study, we improve this point by introducing a kind of feedback system, in which two feedback processes are repeated in an alternating manner.

2. Relationships between Lumped Parameters

In this section, some equations discussed in the previous studies[1-4) are reviewed for later

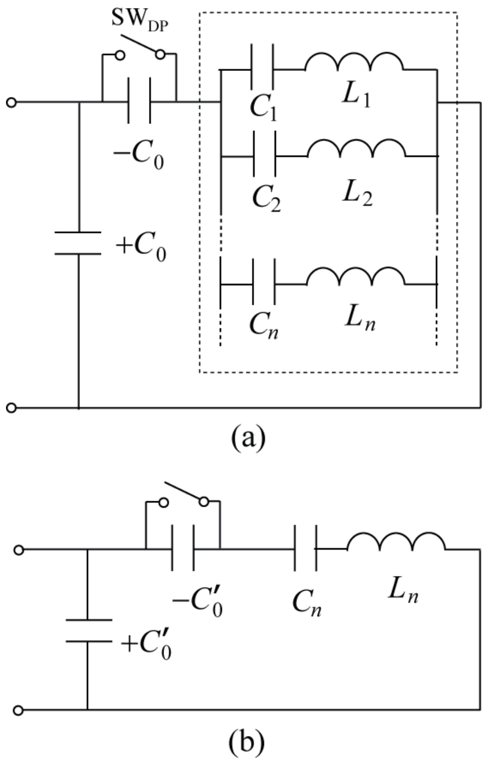

Fig.1 Equivalent circuit: (a) When all modes are considered. (b) When resonance of nth mode is measured. The switch is shorted in the case of transverse (T-) effect, while it is open in longitudinal (L-) effect.

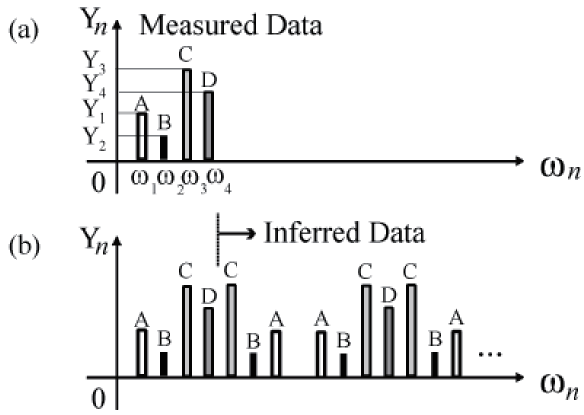

Fig. 2 Resonance pattern in the frequency domain as a pair of ω_n and Y_n. (a) Example of measured data. (b) Inference of pattern in the frequency domain by considering some periodicity in the domain.

discussion. In the frequency (ω-) domain, resonance frequencies and corresponding resonance intensities form a kind of "pattern" with some mathematical regularities, when the observation is performed from ω = 0 to infinity [1,2)]. However, we can usually observe only a finite number of lower resonance modes, as shown in Fig. 1(a), where ω_n is

$$\omega_n = \begin{cases} \omega_A & \text{for L - effect} \\ \omega_R & \text{for T - effect} \end{cases},$$

(for nth mode, where the subscript A and R stand for electrical antiresonance and resonance, respectively), and Y_n is related with L_n in Fig. 1 as

$$Y_n \propto (L_n)^{-1}.$$

In this study, we only set

$$Y_n = (L_n)^{-1}$$

for convenience. We also define

$$\hat{Y}_n \equiv \frac{Y_n}{\omega_n^{\ 2}},\quad \hat{Y} \equiv \sum_{l=1}^{\infty} \hat{Y}_l,$$

(the normalization factor is set unity for the purpose of this study), which equal to C_n and

$$C \equiv \sum_{l=1}^{\infty} C_l ,$$

respectively. The relationship between C'_0 and C_0 in Fig. 1 is given by

$$C'_0 \equiv C_0 \mp g_n C ,$$

which is the *definition* of g_n with mode dependence. (The upper sign for L-effect, and lower sign for T-effect.) From the measurement of the nth mode resonance,

$$r'_n \equiv \left(\Delta\omega_n^{\,2}\right)\times\frac{\hat{Y}}{Y_n} \approx \frac{C}{C'_0}, \quad (\Delta\omega_n^{\,2} \equiv \omega_A^{\,2} - \omega_R^{\,2})$$

$$u_n \equiv \left(\Delta\omega_n^{\,2}\right)\times C'_0 L_n \approx 1 ,$$

are obtained.[3] g_n is estimated conventionally using

$$g_n \approx g_{n(\mathrm{conv})} \equiv \left(\hat{Y} - \sum_{l=1}^{n}\hat{Y}_l\right)/\hat{Y} ,$$

and, in ref. 4, we adopt

$$g_n \approx g_{n(\mathrm{corr})} \equiv g_{n(\mathrm{conv})} + \sum_{l=1}^{n-1}\frac{1}{1-(\omega_n/\omega_l)^2}\frac{\hat{Y}_l}{\hat{Y}} + \sum_{l=n+1}^{\mathrm{higher}}\frac{(\omega_n/\omega_l)^2}{1-(\omega_n/\omega_l)^2}\frac{\hat{Y}_l}{\hat{Y}}$$

from which

$$r_n \equiv r'_n/\left(1 \pm g_n r'_n\right) \approx C/C_0$$

is estimated. r_n corresponds to the capacitance ratio, and therefore, should be invariant regardless of n, which can be utilized for reducing the errors in the measured values of Y_n or L_n^{-1}: Modifying Y_n for multiple modes (n=1,2,…) appropriately, $Y_n \rightarrow Y_n^*$, can make the *variance* of r_n over the measured modes a minimum,

$$\mathrm{Var}[r_n] \equiv \mathrm{Mean}\left[(r_n - \bar{r}_n)^2\right] \rightarrow \min$$

under a constraint condition:

$$\hat{Y} = \text{constant}.$$

This procedure provides more reliable values of Y_n ($\sim L_n^{-1}$) as well as r_n ($\sim C/C_0$).

2. Point of Argument in This Study

In the above method, the value of Y^ must be estimated by inferring the values of ω_n and Y_n in higher modes with the help of the actually measured data on lower modes, as shown in **Fig. 2**. However, this inference process might include some errors due to the following reasons:

(i) The periodicity in the frequency domain on ω_n and Y_n cannot be known in advance.

(ii) The measured data of Y_n in themselves might include some errors *before* the correction mentioned in Sec. 2, from which Y^ is inferred.

Therefore, we introduce some feedback system in this study to reduce the errors in Y^. Note that

$$r'_n\left(\frac{1}{r_n} \mp g_n\right) = \Delta\omega_n^{\,2}\,\frac{\hat{Y}}{Y_n}\left(\frac{1}{r_n} \mp g_n(\hat{Y}, Y_l)\right) \approx 1 ,$$

where g_n is regarded as a function of Y^ and Y_l. With the help of this equation, we introduce

$$U_n \equiv \Delta\omega_n^{\,2}\,\frac{\hat{Y}}{Y_n^{\,*}}\left(\frac{1}{\bar{r}_n} \mp g_n(\hat{Y}, Y_l^{\,*})\right) \approx 1 ,$$

in which $Y_l = Y_l^*$ (l=1,2,…,n,…) are substituted, and U_n is regarded as a function of Y^ and (the mean of) r_n. (However, the error in Y^ influences the mean square error in r_n.[4]) By modifying Y^ and Mean[r_n],

$$\mathrm{Var}[U_n] \equiv \mathrm{Mean}\left[(U_n - 1)^2\right] \rightarrow \min ,$$

which can provide more reliable value of Y^ = Y^*. Using the best estimates at this moment, the above-mentioned processes are repeated, the concept of which is depicted in **Fig. 3**. (Instead of the asterisk (*), the number of times of correction (i) is written as the superscript.)

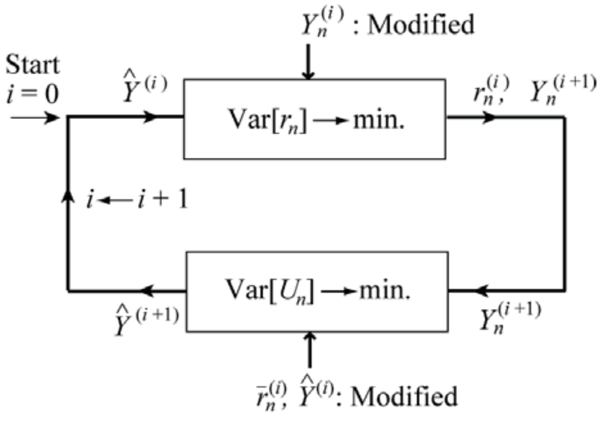

Fig. 3 "Alternating" feedback system to correct Y_n ($\sim L_n^{-1}$), r_n ($\sim C/C_0$), and Y^ ($\sim C$). i is the number of times of the repetitive correction.

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