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Measurement of Young's modulus of Materials Using a Change in Motional Capacitance of the electrical Equivalent Circuit of Quartz-Crystal Tuning-Fork Tactile Sensor at Resonance

音叉型水晶触覚センサの共振時の電気的等価回路における容量変化を用いた物体のヤング率の測定

Hideaki Itoh and Naoki Hatakeyama (Shinshu Univ. Dep. Electr & Elect. Eng)
伊藤秀明，畠山直樹（信州大 電気電子工）

1. Introduction

A tension test for measuring Young's modulus of hard materials such as metals or plastics is generally used. On the other hand, rotary viscometer[1)] and DMA[2)] (Dynamic Mechanical Analysis) are also used to measure Young's modulus of rubbers. However, there are some drawbacks in these equipments. For example, they are so expensive and large, and furthermore, sample measured by them is cut to fit the test piece's sizes because its dimensions are fixed for measuring.

In this study, we focus on the method to measure Young's modulus of materials using a change in motional capacitance of the electrical equivalent circuit of quartz-crystal tuning-fork tactile sensor before and after its base is brought into contact with materials instead of using a change in its resonant frequency. We obtained the relationship between the change in motional capacitance and Young's modulus of one arm of quartz-crystal tuning-fork by use of bimorph flexural model.[3)4)] The experiments were conducted to check whether it would be possible to measure Young's modulus of materials using the change in motional capacitance or not on three sorts of materials such as rubbers, plastics, and metals.

2. Analysis

We analyzed the behavior of quartz-crystal tuning-fork resonator using bimorph flexural model

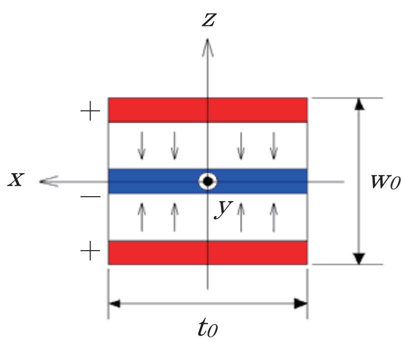

Fig.1.Electrical field lines in cross-section of an arm of quartz-crystal tuning-fork resonator using bimorph flexural model and its coordinates

[3)4)] to derive motional capacitance Ca formula of the electrical equivalent circuit of quartz-crystal tuning-fork resonator at resonance. We apply bimorph flexural model to an arm of quartz-crystal tuning-fork resonator in order to make it easier to analyze. Figure 1 shows the electrical field lines indicated as arrows in cross-section of an arm of quartz-crystal tuning-fork resonator. We set x, y, and z-coordinates as shown in Fig. 1.

The piezoelectric equation for flexural beam shown in Fig.1 is expressed by

$$T_2 = \frac{1}{S_{22}^E} S_2 - \frac{d_{32}}{S_{22}^E} E_3 , \qquad (1)$$

$$D_3 = d_{32} T_2 + \varepsilon_{33}^T E_3 , \qquad (2)$$

where S_{22}^E is compliance coefficient of quartz crystal, S_2 is extensional strain in the y direction, d_{32} is piezoelectric strain coefficient of quartz crystal, T_2 is extensional stress in the y direction, D_3 is electrical displacement, E_3 is electrical field intensity in the z direction, and ε_{33}^T is permittivity of quartz crystal.

When the beam undergoes flexural vibration, extensional strain S_2 and electrical field E_3 is expressed by

$$S_2 = -z \frac{\partial^2 \xi}{\partial y^2}, \quad E_3 = -\frac{\partial \phi}{\partial z}, \qquad (3)$$

where ξ is flexural displacement in the z direction along y-axis and ϕ is potential.

We applied eqs. (1) and (2) to an arm of quartz-crystal tuning-fork resonator as cantilever and derived Ca formula from the input admittance of piezoelectric cantilever at resonance.

According to the mechanical-electrical analogy,[5)] spring constant corresponds to the reciprocal of capacitance. Therefore, the difference in the reciprocal of capacitance $\Delta 1/Ca$ before and after the tactile sensor is brought into contact with materials is given by

$$\Delta \frac{1}{C_a} = \frac{1}{C_a{}'} - \frac{1}{C_a}$$

$$= \frac{1}{12}\cdot\frac{\alpha^2 w_0}{d_{32}^2 l t_0} \times \frac{\cosh\alpha\sin\alpha - \sinh\alpha\cos\alpha}{\cosh\alpha\sin\alpha + \sinh\alpha\cos\alpha}\cdot\frac{E-E'}{E'E}, \quad (4)$$

where C_a' and E' are motional capacitance and Young's modulus of quartz crystal after the tactile sensor is brought into contact with materials, respectively. Eq. (4) is rearranged using A (the constant term in eq. (4)) and $\varDelta E$ ($E-E'$ term in eq. (4)) as

$$\varDelta E = \frac{E'E}{A}\cdot\varDelta\frac{1}{C_a}. \quad (5)$$

3. Experimental results and Discussion

When Young's modulus of materials is much lower than that of quartz crystal, we could adopt rough approximation expression of $EE' \approx E^2$ Therefore, $\varDelta\, 1/Ca$ shows a direct linear dependence on $\varDelta\, E$. However, when Young's modulus of materials is higher than that of quartz crystal, a sign of E' is minus since $E' = E - \varDelta E$ and must be never minus as physical quantity. In these two cases, the behavior of the change in motional capacitance against Young's modulus was different from each other (see Figs. 2 and 3).

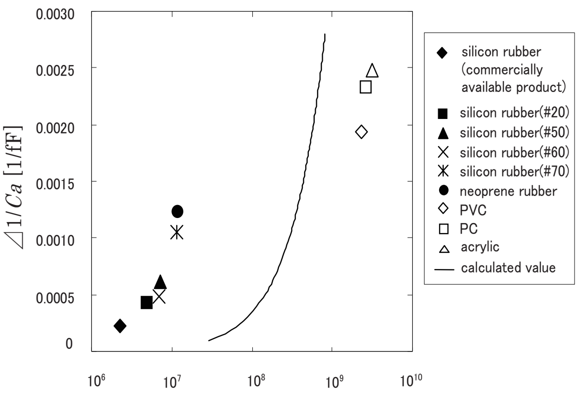

Fig.2 $\varDelta\, 1/Ca$ vs Young's modulus for rubbers and plastics

Figure 2 shows the change in motional capacitance against Young's modulus for rubbers and plastics when their Young's modulus is lower than that of quartz crystal. On the contrary, Figure 3 shows the change in motional capacitance against Young's modulus for metals when their Young's modulus is higher than that of quartz crystal. The solid line in Fig. 2 is the calculated value which is obtained by substituting the measured value of the

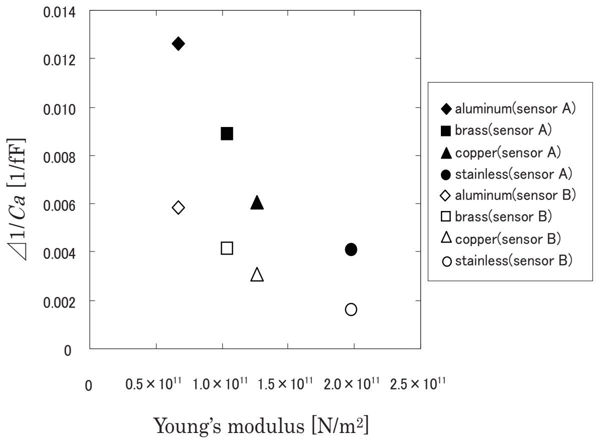

Fig.3 $\varDelta\, 1/Ca$ vs Young's modulus for metals

change in motional capacitance into eq. (5). In Fig. 3, four closed and four open symbols are the measured values using sensor A and B, respectively. The measured values have same tendency for $\varDelta\, 1/Ca$ vs. Young's modulus but the values are not same because two sensors have different sensitivity.

4. Conclusion

We found that there was a possibility to measure Young's modulus of materials using the correlation between the change in motional capacitance of the electrical equivalent circuit of quartz-crystal tuning-fork tactile sensor at resonance and Young's modulus of materials being in contact with its base, and that the behavior of the change in motional capacitance against Young's modulus was very different whether Young's modulus of material was higher than that of quartz crystal or lower.

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