1P2-14

Proceedings of Symposium on Ultrasonic Electronics, Vol. 30 (2009) pp. 43-44
18-20 November, 2009

Group delay method for Evaluation of Layer Material Properties
群遅延時間を用いた薄膜物性の評価

Junjie Chang[1†] and Juichi Nakayama[2] ([1]Japan probe Co. Ltd., [2], Kyoto Inst.Tech.)
常俊杰 [1‡]，中山純一 [2]，（[1] ジャパンプローブ；　[2] 京工繊大 工）

1. Introduction

This paper deals with a group delay method for evaluating thin layer material properties. We give theoretical discussions on the method and some experimental results to demonstrate its feasibility.

In the field of mechanical engineering, thin layer materials are widely used to make machines light in weight and to improve in performance. Thus, evaluations of thin layers are practically important for inspection and maintenance. However, a conventional time domain reflection method using ultrasound pulse wave does not work well for a thin layer, because an echo from one side of the layer overlaps with one from the other side of the layer. In fact, the received echo has a damped oscillation wave form, because multiple reflections take place inside the thin layer and because the thin layer works as a transmission line resonator.

In the group delay method, we obtain, from the spectrum of such a received echo, the group delay, which is a periodic function of frequency and becomes the maximum at resonance frequencies. We point out theoretically that the ratio of the thin layer thickness and the velocity of the layer is determined from the period and the impedance of the thin layer can be obtained from the maximum delay time. Experiments were carried out for a rubber thin layer. Then it is conclude that the group delay method is practically useful for evaluating thin layer material properties.

2. Propagation of ultrasonic wave in thin layer

Let us consider the pulse wave reflection by a thin layer in **Fig. 1**. We write the incident pulse wave $u=u(t-x/v_1)$ as

$$u(t-\frac{x}{v_1}) = \int_{-\infty}^{\infty} \bar{U}(f) e^{-2\pi i f(t-\frac{x}{v_1})} df \quad , \tag{1}$$

and the reflected wave $s=s(t+x/v_1, h)$ in the medium 1 as

$$s(t+\frac{x}{v_1},h) = \int_{-\infty}^{\infty} S(f,h) e^{-2\pi i f(t+\frac{x}{v_1})} df \quad . \tag{2}$$

Here, $\bar{U}(f)$ and $S(f,h)$ are frequency spectra, which are determined from pulse wave forms $u(t)$ and $s(t,h)$ at x=0, respectively, by Fourier transform:

$$\bar{U}(f) = \int_{-\infty}^{\infty} u(t) e^{2\pi i f t} dt \, , \tag{3}$$

$$\bar{S}(f,h) = \int_{-\infty}^{\infty} s(t,h) e^{2\pi i f t} dt \, . \tag{4}$$

Solving the boundary problem, let us obtain $s(f,h)$. Assuming the medium 3 is air and putting $Z_3=0$ approximately, we obtain

$$S(f,h) = -\bar{U}(f)\frac{e^{2ik_2h}-\Gamma_1}{1-\Gamma_1 e^{2ik_2h}} e^{2ik_1x_1} \, , \tag{5}$$

$$k_1 = 2\pi f / v_1, \;\; k_2 = 2\pi f / v_2 \, , \tag{6}$$

$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1} \, , |\Gamma_1| < 1 \, , \tag{7}$$

$$\frac{1-\Gamma_1}{1+\Gamma_1} = \frac{Z_1}{Z_2} \, . \tag{8}$$

Here, k_1 and k_2 are wave numbers, and Z_1 and Z_2 are impedance in media 1 and 2, respectively (see **Fig. 1**). These are all real numbers in a loss free case. Γ_1 is the reflection coefficient by the interface at $x=x_1$. Γ_1 is positive if $Z_2>Z_1$, and it becomes negative when $Z_1>Z_2$.

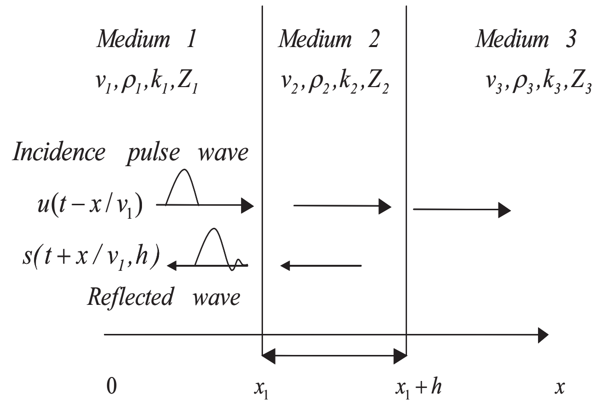

Fig. 1 Pulse reflection by a thin layer (medium 2) with thickness h. v_i, ρ_i, Z_i are the velocity, density and acoustic impedance of medium i, (i=1,2,3), respectively. k_i is the wave number of the medium i. In experiments, medium 1 is acrylic, medium 2 is rubber, and medium 3 is air.

When the thin layer (medium 2) does not exist and h=0, we have form Eq. (5)

$$S(f,0) = -U(f)e^{2ik_1x_1} \, . \tag{9}$$

Next, we define the normalized spectrum by $S(f,h)/S(f,0)$. We find from Eq. (5) and Eq. (9)

$$\frac{S(f,h)}{S(f,0)} = \frac{e^{2ik_2h}-\Gamma_1}{e^{-2ik_2h}-\Gamma_1} e^{-2ik_2h} = e^{i\phi(f)} \, , \tag{10}$$

which becomes a complex number with modulus 1 and phase angle $\phi(f)$ when Γ_1 is real. From Eq.

----

chang@jp-probe.com

(10) we have

$$\phi(f) = -2k_2h + 2\tan^{-1}\left[\frac{\sin(2k_2h)}{\cos(2k_2h)-\Gamma_1}\right]. \quad (11)$$

From this we have the group delay time τ(*f*)

$$\tau(f) = \frac{1}{2\pi}\frac{d\phi}{df} = \left(\frac{2h}{v_2}\right)\frac{1-\Gamma_1^2}{\Gamma_1^2 - 2\Gamma_1\cos(\frac{4\pi fh}{v_2})+1}, \quad (12)$$

which is a periodic function of *f*. The period f_p is

$$f_p = v_2/2h = 1/\Delta\tau. \quad (13)$$

Here, Δτ is the round trip propagation time of the thin layer. Note that f_p and Δτ are independent of Z_1 and Z_2. The group delay τ(*f*) takes the maximum τ_{max} or minimum τ_{min} at a frequency f determined by $\cos(4\pi fh/v_2)=\pm1$. However, there are two cases.

Case1: if $Z_2>Z_1$, then $\Gamma_1>0$, and group delay time τ(*f*) becomes maximum when *f* is an integer times of f_p:

$$\tau(n\cdot f_p) = \tau_{max} = \frac{2h}{v_2}\cdot\frac{Z_2}{Z_1}, \ (n=1,2,3,\cdots) \quad (14)$$

Case2: if $Z_1>Z_2$, then $\Gamma_1<0$, and group delay time τ(*f*) becomes maximum when *f* is an odd integer times of $f_p/2$

$$\tau((2n+1)f_p/2) = \tau_{max} = \frac{2h}{v_2}\cdot\frac{Z_1}{Z_2}, \ (n=1,2,3,\cdots) \quad (15)$$

If *h* is known, we may evaluate v_2 by Eq. (13) as

$$v_2 = 2h\cdot f_p. \quad (16)$$

Also, ρ_2 can be estimated by Eq. (14) and Eq. (15) as

$$\rho_2 = \begin{cases} Z_1\cdot\tau_{max}f_p/v_2, & (Z_2>Z_1) \\ Z_1/(\tau_{max}f_pv_2), & (Z_1>Z_2) \end{cases}. \quad (17)$$

3. Result of an experiment and consideration

In experiments, medium 1 was acrylic (v_1=2700m/s, ρ_1=1200kg/m^3, Z_1=3.24E10Ns/m^3). A thin layer rubber with thickness h=1mm was used as a sample. Using an electric excitation (f_0=1MHz):

$$p(t) = \begin{cases} P_0[1+\sin(2\pi f_0 t-\pi/2)], & 0<t<1/f_0 \\ 0, & \text{else} \end{cases}. \quad (18)$$

We measured echoes *s*(*t*,0) and *s*(*t*,*h*) for the rubber with *h*=1mm. Taking Fourier transform of the echoes, we obtained the group delay τ(*f*) in Fig. 2, in which the period is f_p=0.74MHz and the maximum group delay is τ_{max}=2.56μs. Since the group delay becomes maximum at a frequency *f* equal to an odd integer times of $f_p/2$, it holds that $Z_1>Z_2$. Since h=1mm, v_2 can be estimated as 1480m/s by Eq. (16). Since τ_{max}=2.56μs, ρ2 can be estimated as 1155.6 kg/m^3 by the lower equation of (17). These results are summarized in Table I, where results by a general echo method for a thick sample with h=10mm are shown for comparison. The difference between results by these methods is 6.0% in acoustic velocity and is 5.7% in density. Experiments were carried out for other samples. Comparing with results by a general echo method for a thick sample, we found that the group delay method gives reasonable physical properties for a thin layer material.

Our discussions were restricted to the longitudinal wave case. However, we expect that the group delay method may be applicable for the transverse wave case. Moreover, the elasticity modulus E, the shearing rate G, and the Poisson ratio ν of the material could be estimated from V_L the acoustic velocity and ρ density of the material, if we take the relations $E=\rho V_L^2$ and $G=E/2(1+\nu)$.

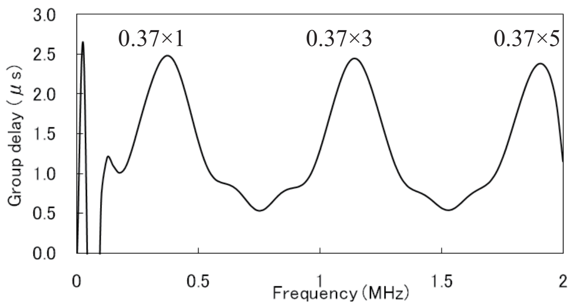

Fig. 2 Group delay τ(*f*) against frequency *f*. rubber layer with *h*=1.0mm.

Table I Comparison of physical property by experiment

Material Series.	General echo measurement		Group delay method		Difference of two methods	
	v_2	ρ_2	v_2	ρ_2	v_2	ρ_2
B4Si	1575	1226	1480	1156	6.0%	5.7%

4. Conclusion

This paper proposed the group delay method for evaluating thin layer material properties. On the base of theoretical discussions for the longitudinal wave, we gave formulas for estimating the sound velocity and density of a thin layer material. Experiments were carried out for rubber thin layers and results were compared with ones by a general echo method for thick samples. Then, we conclude that the group delay method is feasible for estimating thin layer material properties.

References

1. Cox R L, D.P.Almond, H.Reiter. Ultrasonic attenuation in plasma-sprayed coating materials [J]. Ultrasonics, January, (1981)17.
2. L. Brillouim: Wave Propagation and Group Velocity (Academic Press, New York, 1960) 7.
3. L. Rose: *Ultrasonic Waves in Solid Medium*, (Cambridge University Press, London, 1999)24.