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An examination on CIP-ABC for Multi-dimensional FDTD Acoustic Simulation

FDTD音響シミュレーションのためのCIPアルゴリズムに基づく吸収境界条件の一検討

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1. Introduction

Many results using numerical analyses of sound propagation have been achieved by Finite-Difference Time-Domain (FDTD) method [1-4]. For free-space simulation, treatment of the outer absorbing boundary is an important technical issue for FDTD calculation of acoustic fields. Some absorbing boundary conditions (ABCs) have been proposed for FDTD analysis.

To date, Berenger's PML[4] is known to have the highest absorption of outgoing waves. However, that technique requires more memory and more complicated programming than others. On the other hand, Mur's ABC is useful when analyses require no outer boundary which provides extremely high absorption.

This study makes an examination of ABC based on the Constrained Interpolation Profile (CIP) method[5]. This method was proposed recently by Yabe et al. in the field of fluid dynamics. A noticeable feature of the CIP method[6] is that it uses both field values on grid point and their spatial derivatives on grid point, which is a markedly distinctive attribute that distinguishes CIP from conventional methods.

In this study, we attempt to apply the CIP aspect to the ABC for Multi-dimensional FDTD acoustic field analyses. We also report the performance results of CIP-ABC.

2. Formulation

Linear governing equations of acoustic fields are given in Eq. (1) and Eq. (2):

$$\rho \frac{\partial \vec{v}}{\partial t} = -\nabla p \tag{1}$$

$$\nabla \cdot \vec{v} = -\frac{1}{K}\frac{\partial p}{\partial t} \tag{2}$$

In those equations, ρ denotes the density of the medium, K is the bulk modulus, p is the sound pressure, and $\vec{v}$ is the particle velocity. Here, we assume that the calculation is for a lossless medium.

For simplicity, let us examine a two-dimensional model. To analyze two-dimensional (2-D) acoustic field propagation of the x-direction and y-direction, we assume $\vec{v} = (v_x, v_y, 0)$ and thereby obtain the following equations from Eq. (1) and Eq. (2).

$$\frac{\partial v_x}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0 \tag{3}$$

$$\frac{\partial v_y}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial y} = 0 \tag{4}$$

$$\frac{\partial p}{\partial t} + K\frac{\partial v_x}{\partial x} + K\frac{\partial v_y}{\partial y} = 0 \tag{5}$$

Therefore, by applying FDTD algorithm to these three equations, Eq. (6) , Eq. (7) and Eq. (8) are obtained using discretized components of sound pressure and particle velocity on grid points. Here, we use second-order central difference approximation on a staggered grid.

$$v_x^{n+\frac{1}{2}}\left(i+\frac{1}{2}\right) = v_x^{n-\frac{1}{2}}\left(i+\frac{1}{2}\right) - \frac{\Delta t}{\rho}\frac{p^n(i+1)-p^n(i)}{\Delta x} \tag{6}$$

$$v_y^{n+\frac{1}{2}}\left(i+\frac{1}{2}\right) = v_y^{n-\frac{1}{2}}\left(i+\frac{1}{2}\right) - \frac{\Delta t}{\rho}\frac{p^n(i+1)-p^n(i)}{\Delta y} \tag{7}$$

$$p^{n+1}(i) = p^n(i) - K\Delta t\frac{v_x^{n-\frac{1}{2}}\left(i+\frac{1}{2}\right)-v_x^{n-\frac{1}{2}}\left(i-\frac{1}{2}\right)}{\Delta x} - K\Delta t\frac{v_y^{n-\frac{1}{2}}\left(i+\frac{1}{2}\right)-v_y^{n-\frac{1}{2}}\left(i-\frac{1}{2}\right)}{\Delta y} \tag{8}$$

In those equations, Δx , Δy and Δt respectively denote the grid size and the time step; and $f^n(i)$ represents f at the time $n\Delta t$ on grid point $x = i$.

3. Absorbing Boundary Condition based on the CIP Method

This study examines ABC based on the CIP method. Using the derivatives of fields is a unique aspect of this method. Formulation of CIP-ABC is shown in Eq. (7) and Eq. (8). First, we consider v_x on the grid point $i = N_x + 1/2$.

$$v_x^{n+\frac{1}{2}}\left(N_x+\frac{1}{2}\right) = (2\xi^3-3\xi^2+1)v_x^{n-\frac{1}{2}}\left(N_x+\frac{1}{2}\right) - (2\xi^3-3\xi^2)v_x^{n-\frac{1}{2}}\left(N_x-\frac{1}{2}\right) - \Delta x(\xi^3-2\xi^2+\xi)\partial_x v_x^{n-\frac{1}{2}}\left(N_x+\frac{1}{2}\right) - \Delta x(\xi^3-\xi^2)\partial_x v_x^{n-\frac{1}{2}}(N_x-\frac{1}{2}) \quad (7)$$

$$\partial_x v_x^{n+\frac{1}{2}}\left(N_x+\frac{1}{2}\right) = -\frac{6}{\Delta x}\xi(\xi-1)v_x^{n-\frac{1}{2}}\left(N_x+\frac{1}{2}\right) + \frac{6}{\Delta x}\xi(\xi-1)v_x^{n-\frac{1}{2}}\left(N_x-\frac{1}{2}\right) + (3\xi^2-4\xi+1)\partial_x v_x^{n-\frac{1}{2}}\left(N_x+\frac{1}{2}\right) + (3\xi^2-2\xi)\partial_x v_x^{n-\frac{1}{2}}(N_x-\frac{1}{2}) \quad (8)$$

Therein, $\xi = \frac{c\Delta t}{\Delta x}$. In these equations, c represents the sound velocity, i.e. $c = \sqrt{K/\rho}$. Also, $\partial_x = \frac{\partial}{\partial x}$. That is $\partial_x v_x$ represents the spatial derivative of v_x in the x-direction.

In addition, $\partial_x v_x$ is given by Eq. (10).

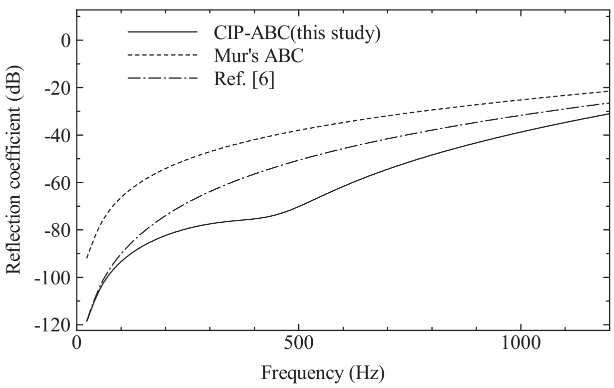

Fig. 1 Reflection coefficient versus frequency

$$\partial_x v_x^{n-\frac{1}{2}}\left(N_x-\frac{1}{2}\right) = \frac{1}{12\Delta x}\left\{2v_x^{n-\frac{1}{2}}\left(N_x-\frac{7}{2}\right) - 11v_x^{n-\frac{1}{2}}\left(N_x-\frac{5}{2}\right) + 27v_x^{n-\frac{1}{2}}\left(N_x-\frac{3}{2}\right) - 17v_x^{n-\frac{1}{2}}\left(N_x-\frac{1}{2}\right) - v_x^{n-\frac{1}{2}}\left(N_x+\frac{1}{2}\right)\right\} \quad (10)$$

4. Numerical Results and Discussion

We show the numerical results obtained using CIP-ABC, as described in section 3. 1-D sound propagation is demonstrated in the analysis domain with CIP-ABC. Now let us consider 1-D model. Calculation parameters are: the direction of acoustic field propagation, $\pm x$ (1-D analysis); grid size, Δx = 0.05m; time step, $\Delta t = 3.657\times10^{-5}$ s; number of grid points, N_x=300; and analysis region, 15m.

Figure 1 shows the results that is calculated using FDTD analysis with present method at x = 12.5m, where ρ =1.21 kg/m^3 and $K = 1.4236\times10^5$. Figure 1 depicts the reflection coefficient of each ABC as a function of frequency.

Next, **Fig. 2** presents the distribution of 2-D acoustic fields with CIP-ABC. This figure validates application of the 2-D acoustic calculation.

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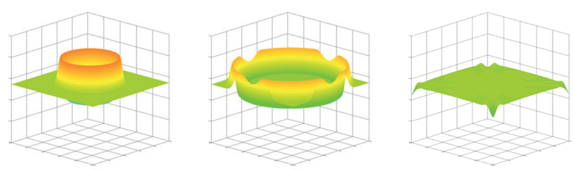

(a) $t = 300\Delta t$ (b) $t = 800\Delta t$ (c) $t = 1200\Delta t$

Fig. 2 Distribution of 2-D acoustic field