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Proceedings of Symposium on Ultrasonic Electronics, Vol. 30 (2009) pp. 29-30
18-20 November, 2009

Numerical Analysis of Piezoelectric Sensor for Simultaneous Measurement of Liquid Density and Viscosity
液体の密度粘度同時計測用圧電センサの数値解析

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1. Introduction

In many factories producing liquids for foods, beautification, or industrial uses, many physical parameters of liquid for example the viscosity or the density are used as an index of the quality control. Usually, one exclusive device is necessary to measure one parameter. It is problem for business management because the dimension of the facility or power consumption becomes huge. To make the facility size small, new devices are developed to measure two or more parameters at the same time. Then, a research to measure the density and the viscosity with one device receives attracts[1-4]. For instance, C. Riesch developed the system that can measure both the density and the viscosity using a longitudinal vibration of the plate at the same time. In this system, the density and the viscosity is presumed by curve fitting of the theoretical characteristics and measured one in the vicinity of the resonance frequency[1]. Thus, the measuring equipment becomes complex.

To improve the function of the measurement device and presume the liquid properties in simple method, we adopt a new method which vibrates the measuring plate toward directions of a longitudinal load and a shear load for liquids. In the proposal system, the liquid density and the viscosity can be presumed by measuring the change in the resonance frequency of the shear and the longitudinal vibrations only. It is expected that the measuring equipment becomes simple and the measurement precision of the proposed method becomes better than that of the conventional method because the frequency can be measured in higher accuracy than frequency response.

In this report, we propose the new method for measuring the liquid density and the viscosity derived from the theoretical model of the vibrating plate toward the longitudinal and the shear directions in a liquid, firstly. Next, the method is evaluated with the Finite Element Method (FEM).

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2. Measurement Principle

The proposal measurement system for the liquid density and viscosity is shown in **Fig. 1**. Two electrodes are attached on each side of the piezoelectric device respectively (i. e. four electrodes are on the whole device). Two electrodes on the upper side are defined as (b) and (c), and those of the reverse side are defined as (a) and (d). To vibrate the plate toward the longitudinal direction, the positive voltage is applied to the electrodes (a) and (d) and the negative voltage is applied to the other electrodes to bend the piezoelectric device. On the other hand, to vibrate the plate toward the shear direction, the electrodes (a) and (b) are applied the positive voltage and the other electrodes are applied the negative voltage to twist the piezoelectric device. The following equation expresses the motion equation showing the sinusoidal forced vibration of the elastic plate in the liquid,

$$F_0 e^{j\omega t} = \left\{k_e - (m_e + m)\omega^2\right\}u + j(b_e + b)\omega u \,. \quad (1)$$

Here, F_0, ω, t, k_e, m_e, b_e and u show the external force amplitude, angular frequency, time, the equivalent mass of the plate, the mechanical resistance, and the spring constant and the displacement of the tip of elastic plate, respectively. Moreover, m and b are the liquid additive mass and the liquid additive damping.

When the plate vibrates to the longitudinal direction, the additive mass m_l and damping b_l have analytical expressions[1,5],

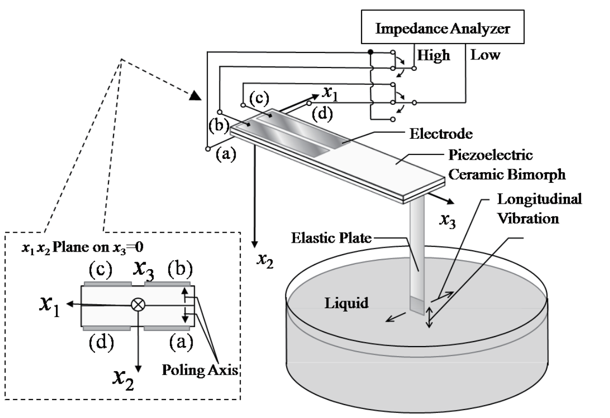

Fig. 1 Proposal measurement system of liquid property

$$m_l = \frac{2\pi R^3}{3}\rho + \frac{3\sqrt{2}\pi R^2}{\sqrt{\omega}}\sqrt{\eta\rho}\ , \qquad (2)$$

$$b_l = 6\pi R\eta + 3\sqrt{2}\pi R^2\sqrt{\omega}\sqrt{\eta\rho}\ , \qquad (3)$$

where, ρ and η are the density and the viscosity of the liquid. Moreover, R is a radius of a sphere in the liquid. The vibration of the plate is approximated by that of the sphere. When the plate is immersed into the liquid at the depth which is same as the width of the plate, R becomes almost the same as the width.

On the other hand, when the plate vibrates to the shear direction, the additive mass m_s and damping b_s have analytical expressions [3-5],

$$m_s = A\sqrt{\frac{1}{2\omega}}\sqrt{\eta\rho}\ , (4) \qquad b_s = A\sqrt{\frac{\omega}{2}}\sqrt{\eta\rho}\ , (5)$$

where, A is the immersed area. The both additive mass m can be obtained from the resonance frequencies in air ω_0 and the changes in the resonance frequency $\delta\omega$ of the shear or the longitudinal vibration, as follows,

$$m = m_e\left\{\left(\frac{\omega_0}{\omega_0 - \delta\omega}\right)^2 - 1\right\}. \qquad (6)$$

Measuring the resonance frequency change in the shear vibration, the product of the density and the viscosity $\eta\rho$ can be derived by Eqs. (4), (6) at first. Next, m_l is obtained from the resonance frequency change in the longitudinal vibration. Then, the density is obtained from Eq. (2). Finally, the viscosity is derived because the product $\eta\rho$ is given already. In a word, the viscosity and the density of the liquid can be presumed by only measuring the change in the resonance frequencies of the shear and the longitudinal vibrations instead of the frequency response. In this method, the equivalent mass of the plate m_e is presumed by measuring the resonance frequency in air and a standard liquid already known the density and the viscosity, because the structure of the sensor device is too complex to derive m_e theoretically.

3. Evaluation with FEM

To ensure the validity of the proposal method, the system is analyzed with Finite Element Method (FEM) including the piezoelectric effect. The virtual stress is applied the tip of the plate as if it is immersed into the liquid. In this condition the resonance frequency is measured by the analysis of the frequency response. Moreover, the measurement precision of the liquid properties in this system is evaluated. The displacement amplitude and the phase at the tip of the plate are shown in **Fig. 2(a)** and **2(b)**, respectively. The dashed, dot-dashed and solid lines show values in the air, the standard liquid, and the sample liquid. Heavy and thin lines show values in the longitudinal and the shear vibrations. At first, the equivalent mass of the device is presumed by measuring the resonance frequencies in the air and the standard liquid. Next, the resonance frequency of sample liquid is measured. Finally, the density and the viscosity of the sample are presumed. The applied and presumed values of the sample liquid are compared in **Table I**. The measurement error is about 7%. A reason of this error is that the resonance frequency to shear vibration hardly change for the frequency resolution. Thus, it is necessary to optimize the design of the device.

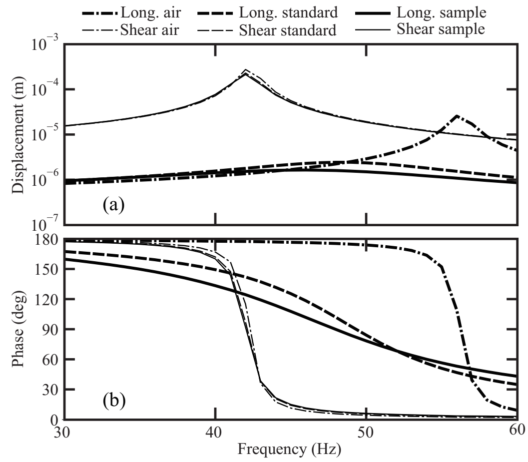

Fig. 2 Frequency response of the plate by longitudinal and shear vibration on liquid: (a) displacement and (b) phase.

Table I Properties of sample liquid used condition of analysis and evaluated by the proposal method

Property	Condition	Measurement
Viscosity (mPa·s)	270	251
Density (kg/m^3)	881	946

4. Conclusion

We propose the new method of measuring the liquid density and viscosity, and evaluate the validity with FEM. In this analysis, there is 7% measurement error because the resonance frequency to the shear vibration hardly changes. It is scheduled to work on the optimum design of the device to improve the measurement sensitivity.

References

1. C. Riesch *et al.*: J. Sensors, **2008**, Article ID 697062.
2. H. Tai *et al.*: Jpn. J. Appl. Phys. **43** (2004) 3088.
3. S. Momozawa *et al.*: Jpn. J. Appl. Phys. **40** (2001) 3654.
4. K. K. Kanazawa and J. G. Gordon II: Analytica Chimica Acta **175** (1985) 99.
5. L.D.Landau and E. M. Lifshitz: *Fluid Mechanics* (Pergamon Press, 1959).