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Modeling of the parametric loudspeakers using the Gaussian-beam expansion technique

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1. Introduction

The parametric array has been widely applied in underwater acoustics, medical ultrasound and nondestructive testing.1) Since the first audio spotlight system was constructed in 1983,2) the development of a directional parametric speaker has attracted much attention. Most of effort has been put into the processing techniques and speaker design to improve the system performance.3-5) Several model equations to describe the propagation of finite sound beams have also been presented.6)

In previous work, the planar piston sources have been used to analyze the parametric array. However, since the parametric loudspeaker comprises multiple small piezoelectric transducers (PZT) or polyvinylidene fluoride (PVDF) film transducers, it's not applicable to assume the uniformly vibrating source to beampattern analysis and array optimization.7) From this point of view, it's necessary to investigate the nonlinear wave propagation for an arbitrary source distribution, especially the transducer array source. In this study, a fast simplified algorithm Gaussian-beam expansion technique is adopted to evaluate the sound fields generated by three source distributions: uniformly rectangular aperture source, arrays with matrix configuration and hexagonal alignment respectively. The axial pressure and beam patterns for primary and second waves are calculated and compared for the three different sources.

2. Theory

The model equation we consider here is the Khokhlov-Zabolotskaya-Kuznetsov(KZK) equation which accurately describes the propagation of finite amplitude sound beams combining the effects of diffraction, absorption and nonlinearity.

$$\frac{\partial^2 p}{\partial z \partial \tau} = \frac{c}{2}\nabla_\perp^2 p + \frac{\delta}{2c^3}\frac{\partial^3 p}{\partial \tau^3} + \frac{\beta}{2\rho_0 c^3}\frac{\partial^2 p^2}{\partial \tau^2} \qquad (1)$$

where p is the sound pressure, z is the axial coordinate along the propagation direction of beams, $\tau = t - z/c$ is the retarded time, c is the small signal sound speed, ρ_0, δ, and β are the ambient density, the dissipation factor related to the sound absorption and the nonlinearity parameter of the medium, respectively.

In the case of weak nonlinearity, the quasi-linear solutions for the fundamental and second-order sound beams are derived using the method of successive approximation. Due to the multiple integrals involved in these expressions, the numerical evaluation is difficult and time-consuming. In order to reduce the computations complexity, the Gaussian-beam expansion technique has been proposed to the calculation of sound beams.8) The calculation expressions presented in ref. 9 are used to evaluate the sound fields with arbitrary source distribution in this study.

3. Numerical Results and Discussion

3.1 The rectangular aperture source and array with elements arranged in matrix configuration

Firstly, the rectangular aperture source and the array with their circle elements arranged in matrix configuration are considered. The side length a of the rectangular aperture source is the physical dimension of the matrix array. According to **Fig. 1**, compact packing is assumed to yield high parametric conversion efficiency. In the simulation, the radius of the circle element is 0.005 m. The matrix array contains 25 elements. The frequencies of two primary waves are 40 kHz and 38 kHz, the pressure of which are both 130 dB. Air temperature is 28ºC and the relative humidity is 60%.

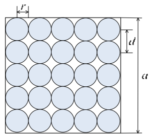

Fig. 1 Diagram of the rectangular aperture source and the matrix array.

The axial sound pressure levels and the farfield directivity on horizontal direction of the primary wave 40 kHz are shown in **Fig. 2**. The axial pressure levels of the primary wave of rectangular aperture source is 2.1 dB higher than that of the matrix array, which equals to the ratio between the area of the rectangular source and the area sum of the 25 transducers used in the matrix array.

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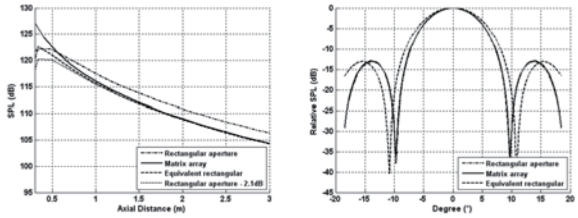
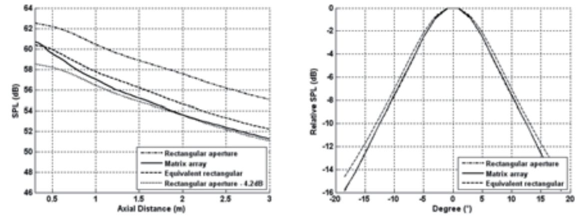
$$20\log10(R) = 20\log10(\frac{S_{rect}}{S_{array}}) = 2.1\,\mathrm{dB} \qquad (2)$$

The dotted line in Fig. 2(a), which is almost the same as the matrix array, represents the sound pressure levels of the rectangular aperture source subtracts 2.1 dB. The sound levels of the equivalent rectangular source which has the same radiation area of the matrix array is calculated as well. As shown in Fig. 2(a), the sound levels of the matrix array and the equivalent rectangular source are almost the same in the farfield, which are about 2.1 dB less than that of rectangular aperture source. The directivity of the equivalent rectangular source is a little broader than that of rectangular aperture source and the matrix array according to Fig. 2(b).



(a) (b)

Fig. 2 The axial sound pressure levels and the directivity on horizontal direction of the primary wave.



(a) (b)

Fig. 3 The axial sound pressure levels and the directivity on horizontal direction of the secondary wave.

Fig. 3 shows the axial sound levels and the farfield directivity on horizontal direction of the difference frequency wave 2 kHz. As shown in Fig. 3(a), the axial sound levels of the rectangular aperture source is 4.2 dB larger than that of the matrix array, which equals to the square of the area ratio between them. The dotted line in Fig. 3(a), which is almost the same as the matrix array, represents the sound pressure levels of the rectangular aperture source subtracts 4.2 dB. The sound levels of the equivalent rectangular source is nearly 1 dB higher than that of matrix array. The directivity of the equivalent rectangular source is almost the same as that of rectangular aperture source and matrix array.

3.2 The arrays with elements arranged in matrix and hexagonal configuration

In order to enhance the parametric conversion efficiency, it's necessary to exploit high packing density configuration in the design of parametric loudspeaker. The comparison between matrix configuration and hexagonal alignment with the same amount transducers are made in this section, as shown in **Fig. 4**. The simulation circumstance is kept being identical as the previous section. The axial sound pressure levels and the farfield directivity on horizontal direction of the difference frequency wave are depicted in **Fig. 5**. The sound levels of the hexagonal alignment array is slightly higher than that of matrix array. But the directivity of both array are nearly the same.

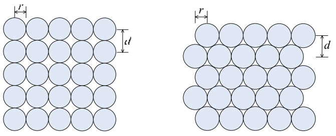
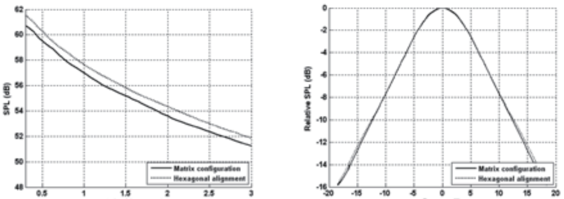


Fig. 4 Diagram of the matrix configuration array and the hexagonal alignment array.



(a) (b)

Fig. 5 The axial sound pressure levels and the directivity on horizontal direction of the secondary wave.

4. Conclusion

The sound beams of the rectangular aperture source and the transducer arrays with matrix configuration and hexagonal alignment are evaluated by the Gaussian-expansion method. For close packing conformation, such as matrix configuration array, the sound levels of the primary and secondary wave is 2.1 dB and 4.2 dB less than that of the rectangular aperture source, which equal to the area ratio and the square area ratio between them, respectively. But the equivalent rectangular source has nearly the equal sound fields of the matrix array. The hexagonal alignment array has almost the same sound beams as the matrix array due to their high packing density.

References

1. M. F. Hamilton and D. T. Blackstock: *Nonlinear Acoustics* (Academic Press, San Diego, 1998) p. 246.
2. M. Yoneyama, J. Fujimoto, Y. Kawamo and S. Sasabe: J. Acoust. Soc. Am. **73** (1983) 1532.
3. T. Kamakura, K. Aoki and Y. Kumamoto: Acustica. **73** (1991) 215.
4. T. D. Kite, J. T. Post and M. F. Hamilton: J. Acoust. Soc. Am. **103** (1998) 2871.
5. F. J. Pompei: J. Audio Eng. Soc. **47** (1999) 726.
6. T. Kamakura: Jpn. J. Appl. Phys. **43** (2004) 2808.
7. J. Yang, W. S. Gan, K. S. Tan and M. H. Er: Jpn. J. Appl. Phys. **44** (2005) 6817.
8. J. Yang, K. Sha and W. S. Gan: IEEE Trans. Ultrason. Ferroelect. Freq. Control. **52** (2005) 610.
9. D. Ding: J. Acoust. Soc. Am. **115** (2004) 35.