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Analysis of guide wave which propagates along pipes filled with fluid
流体を満たしたパイプを伝搬するガイド波の解析

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1. Introduction

Cylindrical pipes are widely used in industries such as nuclear power plants and micro total analysis systems (μTAS). Nondestructive evaluation (NDE) of such pipes is therefore crucial. NDE as well as ultrasonic flowmeters can be used to characterize pipes filled with fluid. In order to apply a guide wave to nondestructive inspection, to know the distribution of the displacement is important because it has influence on sensitivity of detecting defects. Therefore, the author calculated the distribution of the displacement of the guide wave, and verified its result by the finite element method.

2. Theoretical analysis

Fig. 1 shows the theoretical model of a cylindrical pipe and its coordinate system (cylindrical coordinates). The author's theoretical basis is an expansion of that of hollow pipe by Gazis[1]. The displacement $\mathbf{u}^{\text{solid}}$ of the pipe ($a \le r \le b$) and the displacement $\mathbf{u}^{\text{fluid}}$ of a fluid ($0 \le r \le a$) are represented as follows.

$$
\begin{aligned}
u_r^{\text{solid}} &= [f_s' + \frac{n}{r} g_3 - k g_1] \cos n\theta \cos(\omega t - kz) \\
u_\theta^{\text{solid}} &= [-\frac{n}{r} f_s - k g_1 - g_3'] \sin n\theta \cos(\omega t - kz) \\
u_z^{\text{solid}} &= [k f_s - g_1' - \frac{n+1}{r} g_1] \cos n\theta \sin(\omega t - kz) \\
u_r^{\text{fluid}} &= f_f' \cos n\theta \cos(\omega t - kz) \\
u_\theta^{\text{fluid}} &= -\frac{n}{r} f_f \sin n\theta \cos(\omega t - kz) \\
u_z^{\text{fluid}} &= k f_f \cos n\theta \sin(\omega t - kz)
\end{aligned}
\qquad (1)
$$

Here, the author consider acoustic waves that propagate along the z-direction and whose angular frequency is ω and wave number is k. n is the circumferential mode parameter. f_s, g_r, g_θ, g_3 and f_f are represented by a linear combination of Bessel functions.[2] Those coefficients are A, A_1, A_3, B, B_1, B_3 and A_f, and they are decided to satisfy boundary conditions below.

$u_r^{\text{solid}} = u_r^{\text{fluid}}$, $\sigma_{rr}^{\text{solid}} = \sigma_{rr}^{\text{fluid}}$, $\sigma_{r\theta}^{\text{solid}} = \sigma_{rz}^{\text{solid}} = 0$ at $r = a$

$\sigma_{rr}^{\text{solid}} = \sigma_{r\theta}^{\text{solid}} = \sigma_{rz}^{\text{solid}} = 0$ at $r = b$ (2)

$\boldsymbol{\sigma}^{\text{solid}}$ and $\boldsymbol{\sigma}^{\text{fluid}}$ represent the stress tensors of a solid and fluid, respectively. Eq. (2) is represented by a 7 dimensional simultaneous linear equations as follow.

$$
\begin{pmatrix}
c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} & c_{17} \\
c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} & c_{27} \\
c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} & c_{37} \\
c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} & c_{47} \\
c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} & c_{57} \\
c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} & c_{67} \\
c_{71} & c_{72} & c_{73} & c_{74} & c_{75} & c_{76} & c_{77}
\end{pmatrix}
\begin{pmatrix} A \\ A_1 \\ A_3 \\ B \\ B_1 \\ B_3 \\ A_f \end{pmatrix} = 0 \qquad (3)
$$

Here, c_{ij} s are calculated by frequency (f), phase velocity (V) and so on.[2] In order to obtain V, we should seek a phase velocity which satisfies a determinant formula $|c_{ij}| = 0$. In order to obtain displacements, we should solve eq. (3) and substitute the solution into eq. (1).

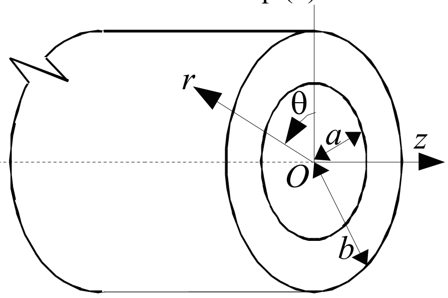

Fig.1 Analytical Model

3. Finite Element Analysis

The author inspected analytical results by axisymmetric finite element method, and the autor used time transient analysis by an explicit method. **Fig. 2** shows finite element mode. Because this analysis is axisymmetric, the author made a rectangle model (hatched area in **Fig. 2**). Displacements were generated by an external stress, and the author simulated a phase velocity scanning (PVS) method[3]. PVS method is one of laser ultrasonic methods, and it can generate a specific acoustic wave which propagates to one direction. In our computer program, a beam width and a pulse width of laser beam are expressed by Gauss functions as below.

$$
\sigma_{rz}^{\text{external}} = \sigma_0 e^{-\frac{4\log_e 2(t-t_0)^2}{9T^2}} e^{-\frac{4\log_e 2(z-z_0)^2}{9\lambda^2}} \sin(\omega t - k(z - z_0)) \qquad (4)
$$

T is period, λ is wavelength. ω, k, T and λ are set as same as a specific guide wave which we wanted to generate. Other parameters are set as follows, $\sigma_0 = 1\text{MPa}$, $x_0 = 80\text{mm}$ and $t_0 = 4.5T/\sqrt{\log_e 2}$. Half pulse width of beam width was set to three times of the wavelength of a guide wave which we wanted to generate, and half pulse width of pulse width was set to three times of the period of a guide wave which we wanted to generate.

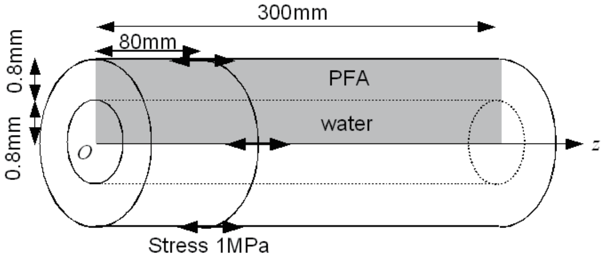

Fig.2 Finite Element Model

4. Results

Fig. 3 shows group velotifies, and **Fig. 4**s show amplitudes of dislacements. Here, "amplitude" means a linear combination of Bessel functions except for sine and cosine of Eq. (1). 102.5kHz is a cut off frequency of a hollow pipe (**Fig. 3**), and whole pipe expands and contracts at this frequency (**Fig. 4**(a)). On the other hand, L(0,2) of a pipe filled with water does not have a cut off frequency (**Fig. 3**). **Fig. 5** shows FEM results of L(0,2)s ($f = 105\text{kHz}$, $V = 1366.5\text{m/s}$ for hollow pipe (**Fig. 5**(a)) or $V = 649.0\text{m/s}$ for pipe filled with water (**Fig. 5**(b))). u_r of a pipe filled with water has a node near $r = 1.1\text{mm}$ (**Fig. 5**(b)), and this result is agreed with the anlaytical result (**Fig. 4**(b) solid curve). **Fig. 5** shows images at the same time ($t = 300\mu\text{s}$), and these images show that a group velocity (v_g) of a hollow pipe is slower than v_g of a pipe filled with water. v_g s calculated by FEM results are appended to the graph (**Fig. 3** ○ and ●). These results are agreed with analytical results.

5. Conclutions

The analytical distributions of the displacement of the guide wave are agreed with FEM results, qualitatively. Dispersion curves are different between a hollow pipe and a pipe filled with water. Therefore, the analysis of a pipe filled with fluid is important for nondestructive evaluation.

References

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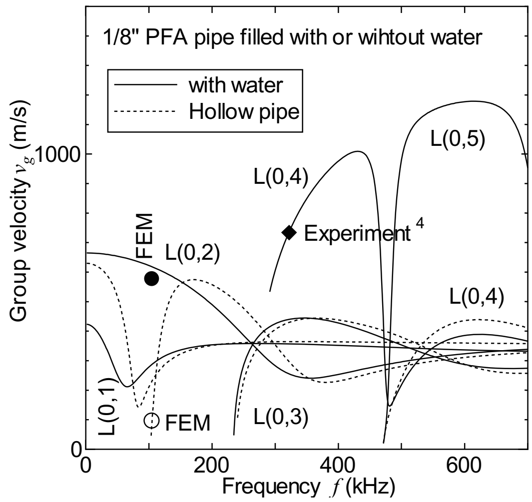


Fig.3 Group velocities

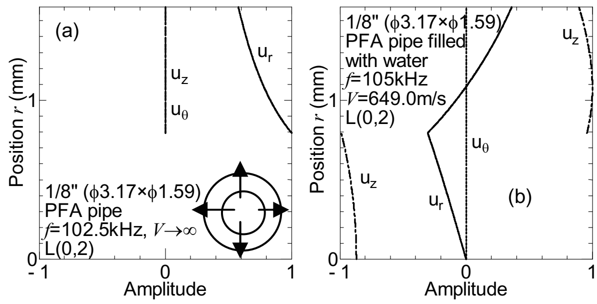
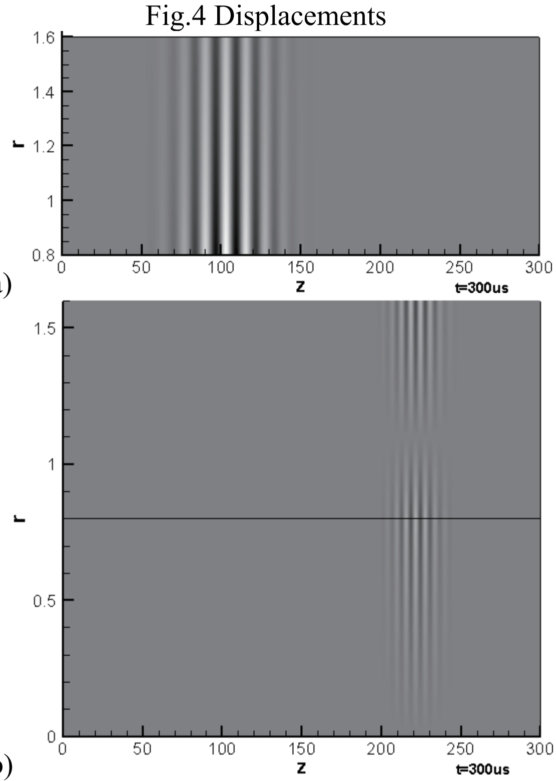


Fig.4 Displacements



(a)
(b)

Fig.5 Intensity modulation images of u_r (105kHz)
(a) Hollow pipe, (b) Pipe with water

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