

# Dispersion of guided waves propagating in a piezoelectric solid and dielectric fluid bi-layer system

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## Abstract

This study is focused in the measurement and modeling for the dispersion relations of guided waves propagating in a piezoelectric solid and dielectric fluid bi-layer system. The theoretical model based on a recursive asymptotic stiffness matrix method (RASM) is used to provide numerical calculations. A laser ultrasound technique is used to measure the dispersion relations. For all the studied cases, the measured dispersion curves agree with the theoretical calculations. Also the effects of fluid loading, including density, thickness, dielectric permittivity constants and conductivity are investigated in a quative way.

**Keywords:** RASM, Lamb waves, laser ultrasound, piezoelectric, dispersion.

## 1.Introduction

While using LiNbO<sub>3</sub> as a substrate material operating in the acoustic plate mode (APM) mode, the dispersion behavior of Lamb waves propagating in LiNbO<sub>3</sub> plates is a fundamental issue and has continuously been studied[1~5]. In particular, in the area of liquid sensor application, the APM devices rely on the mode shifting as the boundary condition is modulated by the ambient fluid.

Previous studies pointed out that both the mechanical and dielectric loading effects are important in the fluid/solid boundary. However, their research is focused on semi-infinity fluid loading, but not the case of a thin fluid layer. It is the purpose of this research to investigate the influence of boundary conditions of a thin fluid layer.

## 2.Theoretical model

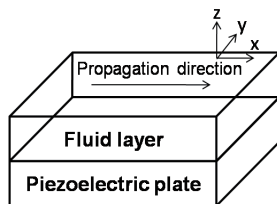


Fig. 1 Bi-layer system and coordinate system.

In this study, a matrix-based theoretical model known as RASM is used to compute the theoretical

dispersion relations of guided acoustic waves propagating in piezoelectric plates loaded with various fluid boundary conditions. The theoretical model for a bi-layer system is shown in Fig. 1.

Following the algorithm of RASM, for a bi-layer system, the dispersion relation function can be represented as the following:

$$G = \begin{bmatrix} S^f + a|k_x|\epsilon_0 S^{fe} S^{ef} & -(1 + \alpha|k_x|\epsilon_0) S^{fe} \\ \alpha S^{ef} & -\alpha S^e \end{bmatrix} \quad (1)$$

With  $\alpha = 1/(1 - |k_x|\epsilon_0 S^e)$ .  $\epsilon_0$  is the vacuum permittivity.  $k_x$  is the wavenumber of guided acoustic wave along X-direction. The  $S^f$ ,  $S^e$ ,  $S^{fe}$  and  $S^{ef}$  mean the function of material matrix of fluid layer and piezoelectric plate.

To simulate the water layer into elastic body which density, wave velocity is similar to water and the shear modular very close to zero. Substituting the material matrix of water layer and piezoelectric plate into equation (1), theoretical calculations of fluid boundary conditions on the dispersion relations of guided waves propagating in piezoelectric plates can be computed.

## 3.Experimental measurement

A laser ultrasound technique is used for the measurement of dispersion spectra of the bi-layer system. Fig. 2 shows the experimental configuration consisting of a pulsed laser for the generation of ultrasonic waves and a laser-based optical detector to detect the generated waves.

In order to obtain the dispersion, a B-scan scheme is used, followed by a two-dimensional fast Fourier transform (2D FFT) scheme to extract dispersion curves.

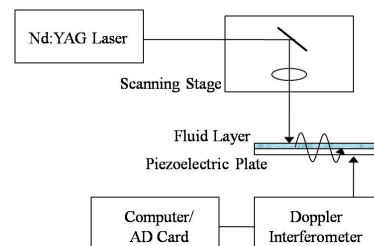


Fig. 2 A schematic showing the measurement system.

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#### 4. Results and discussions

**Fig. 3** and **4** show the dispersion spectra of X-LiNbO<sub>3</sub> in different thickness of water layer-loading. In these figures, cycle symbols represent experiment value, and the solid lines represent theoretical solution. Therefore the experiment is equivalent to theory and identical.

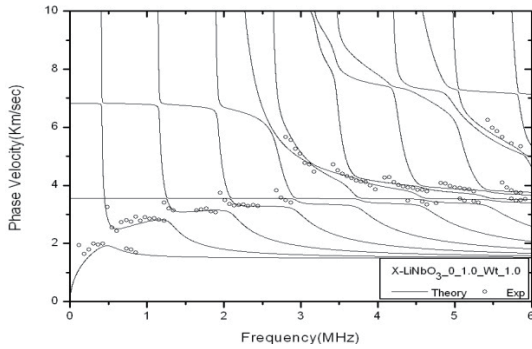


Fig. 3 Dispersion for X-LiNbO<sub>3</sub> with 1mm thick water layer.

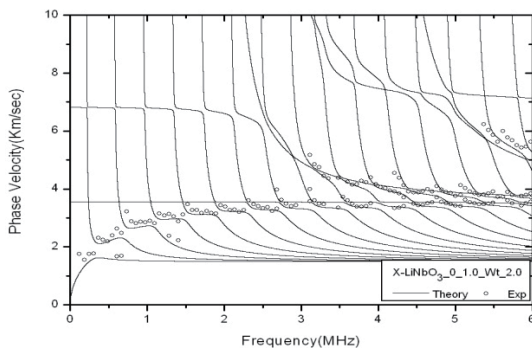


Fig. 4 Dispersion for X-LiNbO<sub>3</sub> with 2mm thick water layer.

Numerical analyses are used to simulate the effect of the mechanical and the electrical boundary conditions of piezoelectric layer/fluid layer bi-layer system on the dispersion relations of guided waves propagating.

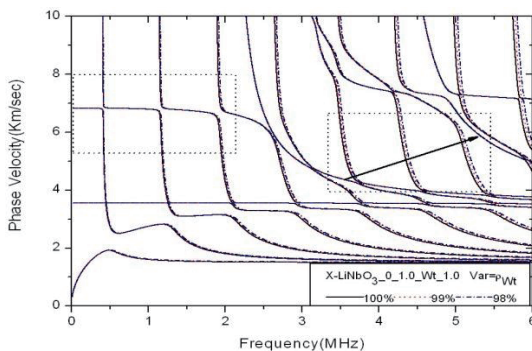


Fig. 5 Dispersion for different densities of fluid loadings.

In mechanical boundary conditions, **Fig. 5** shows the different density ratio between piezoelectric plate and water layer. When the

density of water layer decreases, it represents that the modes shift to high frequency and high velocity. The phenomenon is similar to various thickness and electric boundary conditions, including dielectric permittivity constants and conductivity parameters of water layer.

Especially, in various electric boundary conditions are influenced with different wave propagation direction of piezoelectric plate, namely azimuthal angles or propagation angle. **Fig. 6** is shown that it's more sensitivity in  $\Phi=60^\circ$ . The phenomenon is similar to various conductivity parameters of water layer.

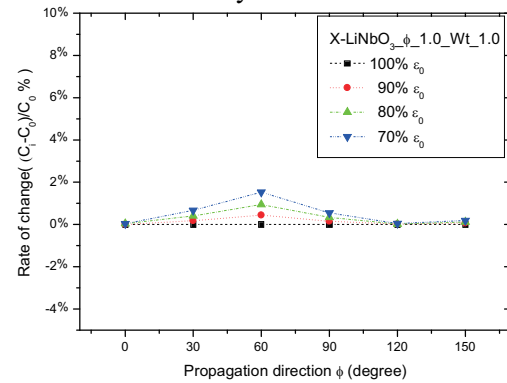


Fig. 6 It presents the rate of phase velocity variations versus different propagation angles with for the various dielectric permittivity constants in 1 MHz.

#### 5. Conclusions

Dispersion relations of guided waves propagating in a piezoelectric solid and dielectric fluid bi-layer system are successfully modeled and verified with experiments. For all the studied cases, the measured dispersion curves agree with the theoretical calculations. The study successfully investigates the effects of fluid on dispersion relation, including density, thickness, dielectric permittivity constants and conductivity. Results of this research provide a useful method and can be useful for the development of Lamb-wave-based sensors.

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