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A Novel Scheme for Numerical Simulation of Acoustic Wave Propagation Using Generalized CIP (*l, m*) Method

GCIP(*l, m*)法を用いた音波伝搬数値シミュレーション

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1. Introduction

To date, numerical analysis for sound wave propagation in time domain has been investigated widely as a result of computer development. Now, the development of accurate numerical schemes is an important technical issue.

The Constrained Interpolation Profile (CIP) method is a novel numerical scheme recently proposed by Yabe[1-6]. It is a method of characteristics (MOC). The feature of the CIP method is that it uses the values of acoustic field and their spatial derivatives at grid points to solve the problem of wave propagation. The family of this scheme is called "Multi-Moment Scheme". While the CIP method has numerical phase velocity with high accuracy for very wide frequency bands, it generates some numerical dissipation error.

The conventional CIP method proposed by Yabe has the third order accuracy in space with the two-point stencils using third–order Hermite interpolation. However, general Hermite interpolation function can realize higher order accuracy by using multi-point stencil.

In this study, we propose generalized CIP (GCIP) method using multi-point stencil and Multi-Moment. We evaluated the calculation performance of the analysis for sound wave propagation by the GCIP (*l, m*) methods.

2. Calculation

The governing equations for linear acoustic fields are given in Eq. (1) and Eq. (2):

$$\rho \frac{\partial \vec{v}}{\partial t} = -\nabla p \, , \tag{1}$$

$$\nabla \cdot \vec{v} = -\frac{1}{K} \frac{\partial p}{\partial t} \, . \tag{2}$$

In those equations, ρ denotes the density of the medium, K is the bulk modulus p is sound pressure and v is the particle velocity. Here we assume that the calculation is for a lossless and homogeneous medium. Moreover, assuming $\vec{v} = (v_x, 0, 0)$ in order to analyze one-dimensional (1-D) acoustic field propagation in the x-direction, we can obtain the following equations from Eq. (1) and Eq. (2).

$$\frac{\partial v_x}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0 \tag{3}$$

$$\frac{\partial p}{\partial t} + K \frac{\partial v_x}{\partial x} = 0 \tag{4}$$

Then, by addition and subtraction of these two equations, we obtain

$$\frac{\partial (p \pm Z v_x)}{\partial t} \pm c \frac{\partial (p \pm Z v_x)}{\partial x} = 0 \tag{5}$$

In those equations, Z indicates the characteristic impedance (i.e. $Z = \sqrt{\rho K}$) and c represents the sound velocity in medium (i.e. $c = \sqrt{K / \rho}$).

In addition, through simple spatial differentiation of the equations, the equations of the derivatives are given as

$$\frac{\partial (\partial_x p \pm Z \partial_x v_x)}{\partial t} \pm c \frac{\partial (\partial_x p \pm Z \partial_x v_x)}{\partial x} = 0 \tag{6}$$

Then, Eqs. (5) and (6) are advection equations of $p \pm Z v_x$ and $\partial_x p \pm Z \partial_x v_x$. Moreover, considering advection of $p \pm Z v_y$ and $\partial_y p \pm Z \partial_y v_y$, we can calculate the propagation in the y-direction as well as in the x-direction.

3. Generalized Hermite interpolation and GCIP (*l, m*) method

The GCIP method uses the general Hermite interpolation to calculate the fields of the (*n* + 1) time step from the value of the discretized fields of *n* time step. Equation (7) shows the method to calculate the fields of the (*n* + 1) with the general Hermite interpolation;

$$
\begin{aligned}
F^{n+1}(x_i) &= \sum_l h_l^{(0)}(x) f(x_l) \\
&+ \sum_l h_l^{(1)}(x) f'(x_l) \\
&+ \sum_l h_l^{(2)}(x) f''(x_l) + \cdots
\end{aligned} \quad . \tag{7}
$$

In those equations, $h_l^{(m)}$ is the interpolation function for the m^{th}-order derivative. l is the number of support-points. The GCIP (l, m) method employs the derivatives to the m^{th}-order.

4. Numerical results

We implement the 2-D acoustic field analysis using the GCIP method. The calculation performance of the 2-D analysis was evaluated for sound wave propagation by the proposed methods. Analytical parameters of calculations were grid size, dx, dy = 0.05 m; time step, dt = 0.05 ms; and an air medium. The CFL number was found to be 0.343.

First, **Fig. 1** shows the normalized numerical phase velocity versus azimuthal angle at 730 Hz. (i.e., point per wavelength; PPW = 9.4.) This clarifies that slight phase error of the GCIP result is generated, while the FDTD method provides lower accuracy. The result of GCIP (7, 1) analysis agrees well with that of GCIP (5, 2) analysis. (These lines are overlapped.)

Next, **Fig. 2** shows the numerical dissipation (dB / λ) versus azimuthal angle at 730 Hz. The numerical dissipation in the GCIP (7, 1) and GCIP (5, 2) and FDTD analyses is little generated. However, conventional CIP, which is corresponding to GCIP (3, 1), does dissipate numerically. In the CIP analysis, the numerical dissipation of the acoustic fields becomes large at the azimuthal angle of 0 deg.

4. Conclusion

This study proposes generalized CIP method for the numerical simulation of acoustic wave propagation. We examine the accuracy of acoustic field analysis using the proposed method, as well as the influence of the propagation angle and phase properties. As a result, this study demonstrates that the GCIP method with multi-point stencil can analyze acoustic field propagation with high accuracy.

In this paper, a comparatively simple analytical model is used as an introduction of the research on acoustic field analysis by the GCIP method. We intend to investigate treatment of a more realistic model in the near future.

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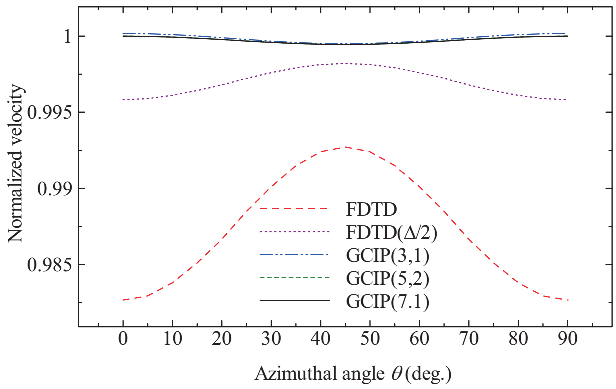

Fig. 1 Normalized phase velocity versus azimuthal angle. (PPW = 9.4.)

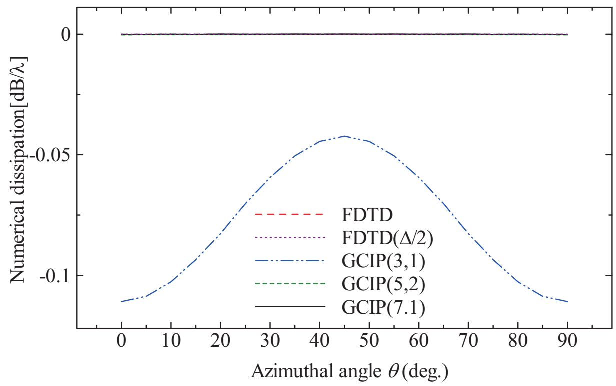

Fig. 2 Numerical dissipation versus azimuthal angle. (PPW = 9.4.)