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Numerical Simulation of Ultrasound Propagation in Inhomogeneous Medium Including Randomly Moving Scatters

ランダム運動を伴う散乱体を含んだ不均質媒質中の超音波伝搬シミュレーション

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1. Introduction

Sound propagations and scatterings in time-dependent inhomogeneous media are interesting from a basic physical standpoint, and they also occur in applied sciences and technologies, for examples, radar, sonar, nondestructive testing, geophysical prospecting and ultrasonic medical diagnosis. Since natures of these problems are more complicated than that in a time-independent inhomogeneous medium, an analysis technique of wave propagations in the time-dependent media, in particularly in that with comparable medium evolution time to wave propagation time, has not reached a well-developed stage for analyses.[1-3)]

In this study, to investigate an interaction between time-dependent inhomogeneous material and ultrasound propagations, we propose a model of ultrasound propagation in a medium including randomly moving scatters as an effect of the time-dependent material, and carry out numerical simulations of ultrasound propagations based on the proposed model.

2. Simulation Model

To simplify the problem, we analyze wave propagations in a two-dimensional (x, y) model. Consider the situation in which a scatter layer with the thickness L_s in the host medium with size of $L_x \times L_y$. In the scatter layer, the number density n_s per $(10\lambda_0)^2$ of scatters are uniformly distributed at the initial step, where λ_0 means the center wavelength of transmitted wave in the host medium (see **Fig. 1**). The statistical property of scatter size is characterized by Gaussian distribution with the mean diameter d_{s0}.

The physical properties of material are characterized by the density ρ and sound speed c. To represent a material at a location (x, y), we introduce the scatter density function $\phi_s^{(m)}$. $\phi_s^{(m)}$ has zero value for outside of m-th scatter (m=1, 2,$\cdots$, M is a scatter number, and M is the total number of

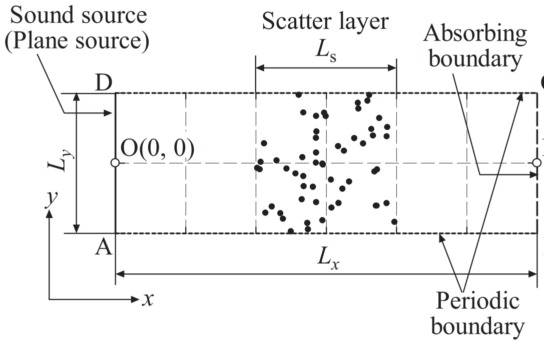

Fig. 1. Analytical model of sound propagation through inhomogeneous medium.

scatters), that is the host medium. On the other hand, $\phi_s^{(m)}$ has unity for inside of m-th scatter. Using the scatter density function, the density ρ and sound speed c are determined by following functions:

$$\rho = \sum_{m=1}^{M} \left[\left(1 - \phi_s^{(m)}\right)\rho_0 + \phi_s^{(m)}\rho_s \right], \tag{1}$$

$$c = \sum_{m=1}^{M} \left[\left(1 - \phi_s^{(m)}\right)c_0 + \phi_s^{(m)}c_s \right], \tag{2}$$

where, ρ_0 and c_0 are the density and sound speed of the host medium, respectively, and ρ_s and c_s are those of the scatter material, respectively. The change of distributions of scatter density function with time provides a time-dependent material.

The sound pressure p and particle velocity $\boldsymbol{u}$ in time domain t are governed by the following equations:

$$\frac{\partial \boldsymbol{u}}{\partial t} = -\frac{1}{\rho}\nabla p + \sum_{m=1}^{M} \left[-\alpha^{(m)}\left(\bar{\boldsymbol{u}} - \boldsymbol{U}_s^{(m)}\right) + \boldsymbol{F}_{\mathrm{ext}}^{(m)} \right]\phi_s^{(m)}, \tag{3}$$

$$\frac{\partial p}{\partial t} = -\kappa \nabla \cdot \boldsymbol{u}, \tag{4}$$

$$\alpha^{(m)} = \frac{1}{2M_s^{(m)}}\rho_0 \left|\bar{\boldsymbol{u}} - \boldsymbol{U}_s^{(m)}\right| d_s^{(m)} C_d, \tag{5}$$

$$\bar{\boldsymbol{u}} = \frac{1}{S_s^{(m)}} \int \boldsymbol{u}' \phi_s^{(m)} \mathrm{d}S, \tag{6}$$

where $\kappa=\rho c^2$ is the volume elasticity, $\boldsymbol{F}_{\mathrm{ext}}^{(m)}$ is an external force acting on m-th scatter per unit mass. $d_s^{(m)}$ is the m-th scatter diameter, $M_s^{(m)}=\rho_s S_s^{(m)}$ is the

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mass of m-th scatter per unit length, and $S_s^{(m)}$ is the cross sectional area of m-th scatter. In addition, $C_d=8\pi/(ReS)$ is the coefficient of viscous drag force for a infinite column in a viscous fluid, Re is the Reynolds number, $S=1/2-C-\ln(Re/8)$, and $C=0.57721$ is Euler's constant.[4]

The velocity $\boldsymbol{U}_s^{(m)}$ of m-th scatter is defined by an average of velocity on the area $S_s^{(m)}$

$$\boldsymbol{U}_s^{(m)} = \frac{1}{S_s^{(m)}} \int \boldsymbol{u} \phi_s^{(m)} \mathrm{d}S \, . \tag{7}$$

For a driving force for random scatter motion, the two-dimensional Gaussian distribution random force with the mean value of zero and the standard deviation σ is consider as the external force $\boldsymbol{F}_{ext}^{(m)}$.

Sound propagations given by Eqs. (1)–(7) are simulated using a finite-difference time domain method based on a staggered mesh system and a leap frog algorithm.

Suppose that materials are uniform and at rest $t<0$. A plane wave source driven by a Gaussian envelope pulse train $p_0(t)$ as shown in **Fig. 2**(a) with the maximum pressure amplitude P_m, center frequency $f_0=\omega_0/(2\pi)$ and repeat period duration T is applied to the boundary $\overline{AD}$ in Fig. 1. Boundaries $\overline{AB}$ and $\overline{CD}$ are characterized by periodic boundaries to represent inward and outward waves. To avoid sound reflections from the opposite side of the source, an absorbing boundary condition is applied to the boundary $\overline{BC}$.

3. Examples of Simulation Results

Examples of simulated pressure wave p_t at three different evolution times 360 μs apart measured at P in Fig. 1 are plotted in Fig. 2(b) and (c). The host medium and the scatter material are assumed to be water ($\rho_0=1.0\times10^3$ kg/m^3, $c_0=1.5\times10^3$ m/s) and an acrylic resin ($\rho_s=1.2\,\rho_0$, $c_s=2.7c_0$), respectively. The size of the analysis area is $L_x/\lambda_0=60$ and $L_y/\lambda_0=20$, and a scatter layer with the thickness of $L_s/\lambda_0=20$ contains scatters of the number density $n_s=10$ which has the mean diameter $d_{s0}/\lambda_0=1$. The sound source is set to $P_m=1$ kPa, $f_0=1$ MHz, and $T=2L_x/c_0=120$ μs. Random force conditions are set to $\sigma/(\omega_0 U_0)=0$ and 10^{-1}, where $U_0=P_m/(\rho_0 c_0)$.

Waveforms of scattered waves are similar for earlier propagation times (the left part of each panel). However, as the scatters move, the waveforms indicate fluctuations of phase and amplitude for later propagation times in the right parts for $\sigma\neq0$ in (c).

These fluctuations are caused by relative changes of sound paths between scatters. This result suggests that quantitative evaluations of fluctuations in scattered waves can measure relative motions of scatters in homogeneous media.

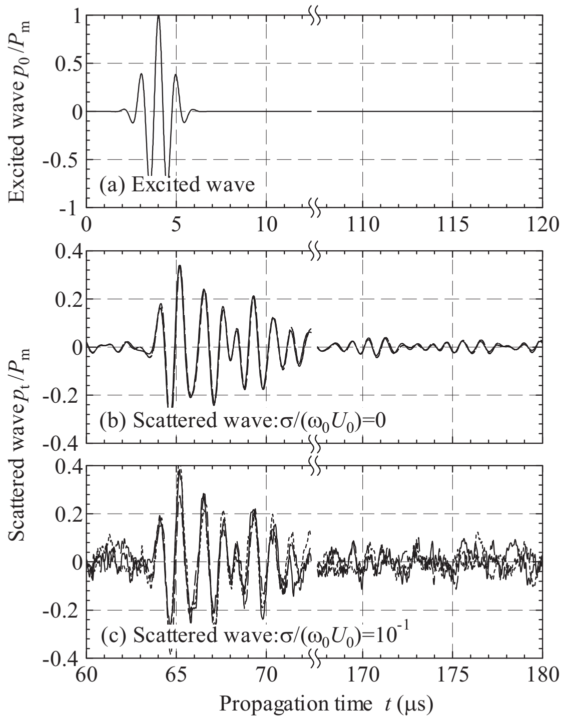

Fig. 2. Excited wave p_0 (a) and scattered waves p_t through a scatter layer at three different evolution times 360 μs (b) and (c).

4. Conclusion

The present work intended to construct a numerical model for analysis sound propagations through time-dependent inhomogeneous medium including randomly moving scatters and to numerically simulate sound propagations based on the proposed model. The result indicates that random motions of scatter increase fluctuations of amplitude and phase in propagation waves.

For future work, we will attempt to construct a numerical model including effects of nonlinear acoustics, such as the acoustic streaming and radiations force.

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