

## Distributed-Parameter Based Treatment of Interaction Process between Elastic and Dielectric Energy in Piezoelectric Transducer: Part II

弾性／誘電エネルギーの圧電振動子内での相互作用の分布定数的取り扱い : Part 2

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### 1. Introduction

The characteristics of piezoelectric transducer can be represented using lumped parameters, which is useful when the (global) characteristics in the transducer as a whole are observed. From the local point of view, however, the characteristics should be expressed with distributed parameters. When the coupling phenomenon between dielectric and elastic components in the transducer is considered, the previous equivalent circuit models generally

include some lumped parameters; for example, in Mason's circuit, the existence of capacitance components  $C_0$  is inevitable, by which the shift of resonance frequency when the coupling occurs can be expressed. (See Fig. 1 with regard to the shift of resonance frequency, where  $\omega_R$  and  $\omega_A$  are electrical resonance and antiresonance frequencies, respectively.)

We have already treated the shift of resonance frequency due to the piezoelectric coupling phenomenon only from the distributed-parameter point of view, when the system is constructed with only *one* layer; that is, not with multiple layers.<sup>1)</sup> We have also treated the calculation method of characteristic functions in a multiple-layered transducer on a distributed-parameter basis, when this coupling phenomenon does *not* occur.<sup>2,3)</sup>

In this study, the treatment of this coupling phenomenon is pursued along our methodology in the case of multiple-layered system.

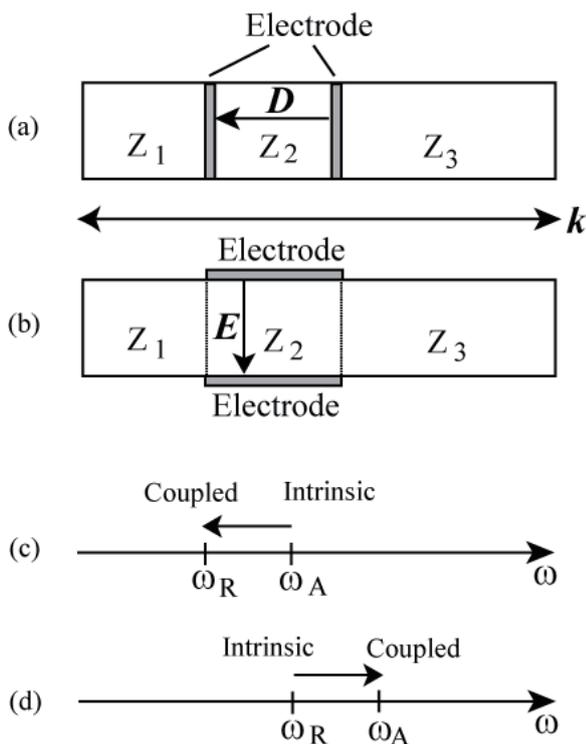


Fig. 1 (a), (b) Piezoelectric transducers with mechanical impedance mismatch layers, partially driven piezoelectrically: (a) In the case of longitudinal (L-) effect; (b) In the case of transverse (T-) effect. (c), (d) Shift of resonance frequency from an “intrinsic” state to a “coupled” state: (c) In the case of L-effect; (d) In the case of T-effect.

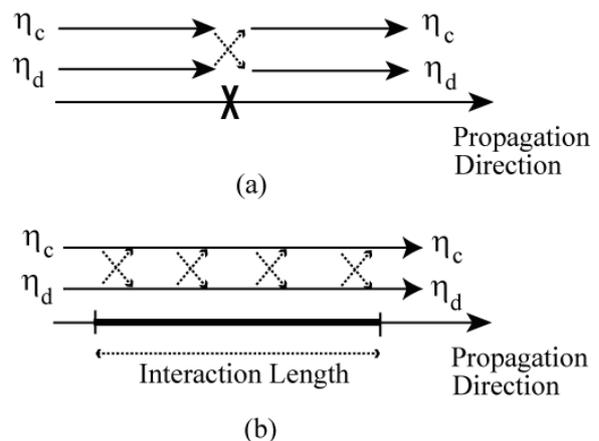


Fig. 2 Interaction between two types of energy modes,  $\eta_c$  (elastic mode) and  $\eta_d$  (dielectric mode) from the distributed-parameter point of view. (a) In the case of “point interaction”. (b) In the case of spatially “continuous interaction”.

## 2. Formulation of Interaction Process

In order to express the coupling phenomenon mentioned above, two types of energy modes are introduced:  $\eta_c$  for “elastic mode” and  $\eta_d$  for “dielectric mode”, as shown in **Fig. 2**. Both modes can propagate in the transducer, and may interact with each other according to the boundary conditions for the system. Figure 2(a) shows the interaction at one spatial point labeled “X”, and Fig. 2(b) shows the case in which the interaction occurs in a spatially *continuous* manner along the region depicted with a bold line with some interaction length. Let us call the former “point interaction” and the latter “continuous interaction”.

In the case of Fig. 2(a), the interaction process is represented with a matrix operation using unitary matrix,  $U_{\text{point}}$ , which expresses an energy conservative process. On the other hand, in the case of Fig. 2(b), not only the energy exchange process but the propagation of the modes (with phase shift and amplitude attenuation) must be considered in a mixed manner, and the matrix for this process is constructed with an “exp” (exponential) of the sum of “log” (logarithm) of two types of matrices for the propagation process  $D_{\text{prop}}$  (non-unitary matrix) and for the energy conservative process  $U_{\text{continuous}}$  (unitary matrix).<sup>1)</sup>

These energy modes are actually confined in the transducer, repeating reflection at the edges of a system or some impedance mismatch boundaries

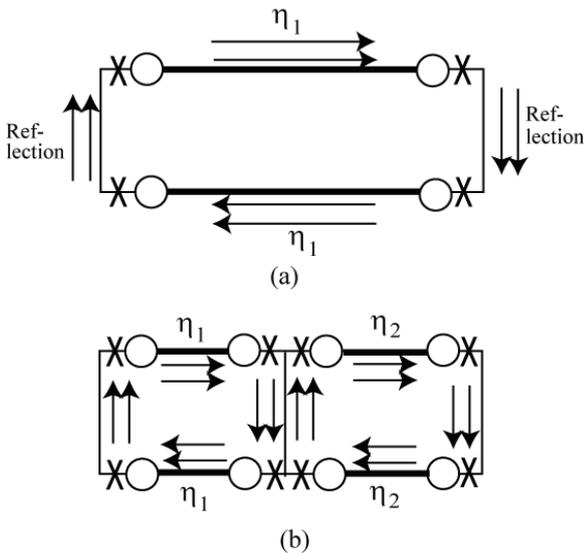


Fig. 3 Treatment of the confinement of energy modes inside the piezoelectric transducer when the coupling phenomenon occurs. (a) In the case of a system with one layer. (b) In the case of a system with two layers.

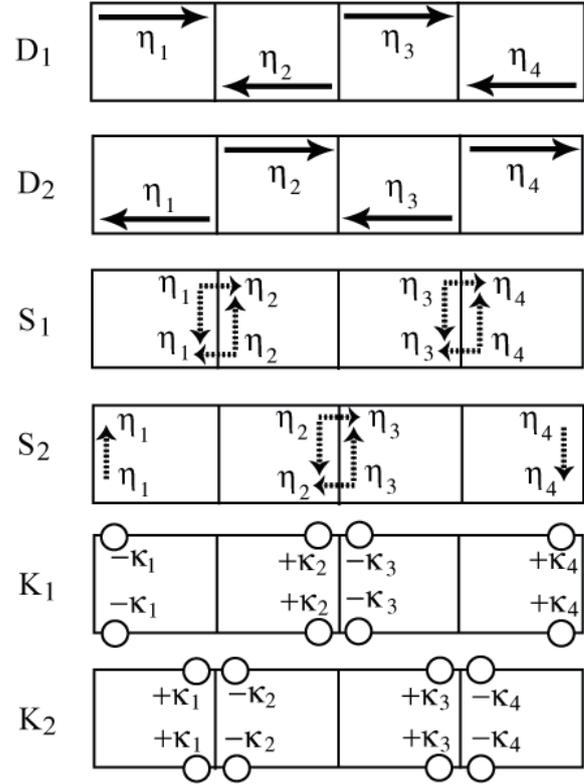


Fig. 4 Six types of matrices to express the behavior of energy modes (in the case of four-layered system, for example).  $D_1$ ,  $D_2$ : for propagation;  $S_1$ ,  $S_2$ : for reflection and transmission;  $K_1$ ,  $K_2$ : for input and output processes.<sup>2)</sup>

inside a multiple-layered system. We introduce a circuit model, as shown in **Fig. 3**, in which these two modes propagate (expressed with two parallel arrows) and interact with each other. Input and output processes for the modes are performed at the positions depicted with open circles ( $\circ$ ). The continuous and point interaction are performed on the bold lines and the positions labeled “X”, respectively. The confinement of these modes causes a kind of superposition, which is represented with infinite geometric series of matrices, “Neumann series”, providing characteristic functions of the transducer. Matrix components for the construction of the series are illustrated in **Fig. 4**, which has been introduced in ref. 2, and in this study, the expression for the point interaction is merged into matrices  $S_1$  and  $S_2$ , and the expression for the continuous interaction is included in matrices  $D_1$  and  $D_2$ . The elements in unitary matrices  $U_{\text{point}}$  and  $U_{\text{continuous}}$  are determined by the boundary conditions of the system.

## References

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3. M. Ohki: Jpn. J. Appl. Phys. **46** (2007) 4474.