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Plate Waves in Locally Resonant Sonic Materials

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1. Introduction

In the past decade, exceptional properties of elastic wave propagation were found to exist in the man-made periodic heterogeneous structures due to the characteristics of multiple scattering.[1-3] Such a kind of elastic structures is called *phononic* or *sonic crystals*, in particular, giving rise to strong sound attenuation if the wavelength is in the order of the periodicity of the structure (attributed to the Bragg scattering). In the structure, the frequency ranges where the sound is strongly attenuated are referred to as phononic band gaps. The phenomenon of band gap, hence, gives prossible application to guiding and filtering of acoustic waves, shielding of noise or vibration, etc. However, such a band gap is not practical in attenating the low-frequency sound or vibration (in the range of tens to hundreds of hertz) because the structure will be unacceptably large.

To use the sonic crystals as sound or vibration barrier in the low-frequency range practically, the band-gap frequencies have to be reduced by at least one or two orders of magnitude lower than those induced by the Bragg scattering in the structures of comparable size. This can be realized ultilzing sonic crystals with locally resonant structural units, called locally resonant sonic materials (LRSM).[2,3] LRSM contains arrays of elastic resonators composed of a high-density solid core with a coating of a very soft material (silicone rubber) or a very soft continuum unit alone.[3] The weak sitffnesses of rubber induce low-frequency resonance, and thus, reduce the band gap frequencies. However, existing investigation on LRSM focused on the propagation of bulk waves, in spite of the wide applications of plate structures and plate waves in engineering.

In this paper, the plate waves in LRSM that consists of an array of silicone rubber resonators in an flat epoxy plate are theoretically studied. For the analysis, a finite element (FE) method is employed to calculate the band structure (i.e., the dispersion relations), eigenmodes, and attenuation speatrum of the plate wave propagation in the LRSM. The local resonances are found, and the plate modes in the LRSM exhibit low-frequency band gaps in the range of hundreds hertz, which can be useful for vibration shielding in engineering utilizing the plate structures. The lattice spacing of the LRSM is two order of magnitude smaller than usual sonic crystal to have a band gap in the same frequency range.

2. Dispersion Relations in LRSM Plates

Schematics of a unit cell of a square lattice LRSM plate is shown in **Fig. 1**. The LRSM plate is assumed to be infinite and periodically arranged in the x and y direction. The matrix is epoxy and the filling material is silicon rubber. The lattice spacing a and the radius r of the circular rubber are 10mm and 8mm, respectively, and the plate thickness h is 1.15mm. To calculate the frequency band structure of acoustic wave propagating in the structure, the COMSOL MULTIPHYSICS[4] is used to carry out the FE calculations. In the FE model, unit cell as Fig. 1 is constructed and meshed. According to the Bloch theorem, the displacement fields obey the following condition on the boundaries of the unit cell:

$$u_i(x+a, y+a) = u_i(x, y)\exp(ik_x a + ik_y a) \qquad (1)$$

where k_x and k_y are the components of the Bloch wave vectors in the x and y directions, respectively. **Figure 2** shows the calculated band structure by the FE method. In the calculations, the mass densities for the epoxy and silicon rubber are 1180 and 1300 kg/m^3, respectively. Longitudinal and shear wave velocities in epoxy are c_L=2534 m/s and c_T=1147 m/s, respectively. In silicon rubber, c_L=33 m/s and c_T=5 m/s, respectively. Figure 2 exhibits a complete band gap in the frequencies as low as from 0.808 to 0.878 kHz. It can be observed form the figure that the band structure is a result of the interaction of dispersion curves of plate waves in the epoxy with the resonance of the circular rubbers. The resonance frequencies of the rubbers are relevant to the flat band frequencies of the band structure, and curves of high slopes near the Γ point correspond to the dispersion curves of the plate waves in the epoxy. They can be identified as the A0, S0, and T0 (i.e.,

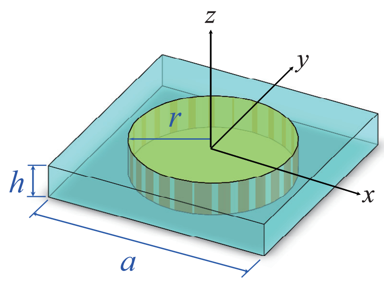

Fig. 1 Unit cell of the square lattice LRSM plate.

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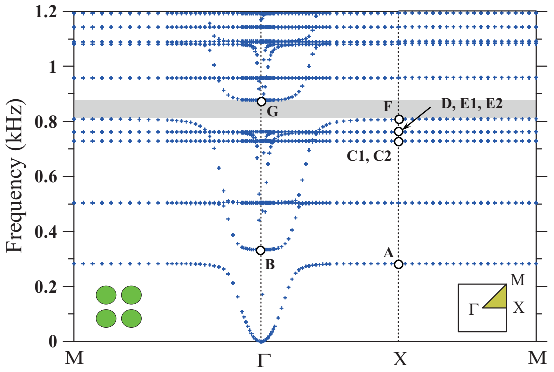

Fig. 2 Frequency band structure of plate waves in the LRSM plate. Gray area denotes the complete band gap.

the lowest antisymmetric, symmetric and transverse) modes in the epoxy plate. Moreover, there exist several flat bands extending the whole Brillouin zone boundary ΓXMΓ. These flat bands are also resonant modes; however, they do not interact with the plate wave modes in the epoxy. Therefore, it is not efficient to induce local resonances in rubbers as waves are incident from the epoxy.

3. Modal Analysis of Local Resonances

To understand the vibration and resonance in the LRSM plate, displacement fields of several eigenmodes (labeled on Fig. 2) are illustrated in **Fig. 3**. In the figures, modes D, E1 and E2 are in-plane modes localized in the rubber, and others modes are all flexural modes whose polarizations are mainly out-of-plane. Notice the gap-edge modes F and G, the upper and bottom edge modes F and G of the complete gap have the same resonance patterns but out of phase. This property also can be found for the modes A and B. Though there is no complete gap between these two modes; however, it is expected that there could be selected gap if the incidence is only flexural, and this gap is at a much more low-frequency range. Modes C1 and C2 have similar patterns and close eigenfrequencies. A more interesting observation is that the dispersion curves of the modes C1 and C2 extend whole Brillouin zone boundary, means that they are not excitable by incident source launched from epoxy.

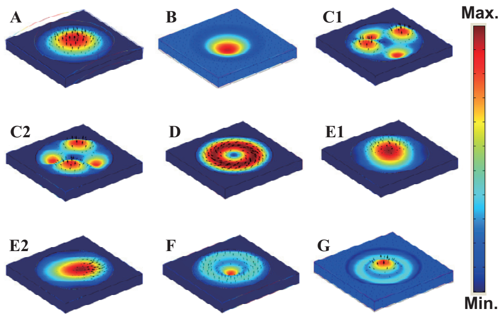

Fig. 3 Displacement fields of several resonant modes in the frequency band structure.

4. Attenuation Spectrum

To evaluate the attenuation spectrum of plate waves propagating through a finite LRSM plate, a structure contains eight unit cells is analyzed. The out-of-plane force is applied to excite acoustic wave at difference frequencies. The energy responses of plate waves propagating through the LRSM plate are compared with those in the homogeneous epoxy plate. The result is shown in **Fig. 4**. It can be clear seen that plate waves attenuate significantly in the complete band gap predicted by the band structure. Moreover, an additional gap in the lower frequency range is also observed from this spectrum. This gap coincides with the range between modes A and B, which shares the same property as the complete gap and is expected when the excitation is out of plane.

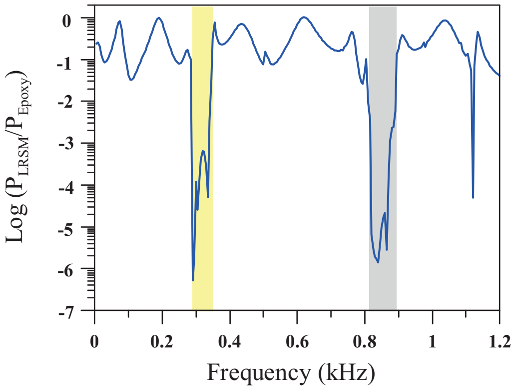

Fig. 4 Attenuation spectrum of plate waves through an eight-cell LRSM.

5. Concluding Remarks

We theoretically analyzed the propagation of plate waves in the LRSM, complete band gaps are found in the frequency range two orders of magnitude smaller than those in Bragg type sonic crystals. The attenuation spectrum of plate waves in finite size LRSM agrees well with that predicted by the band structure.

Acknowledgment

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