

## Observation of Bragg and Hybridization Gaps in 2D Phononic Crystals

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The propagation of classical waves through strongly scattering materials continues to attract interest, largely because the nature of wave transport in such materials may be dramatically altered. Much of the recent interest has focused on ordered materials such as photonic and phononic crystals, which may exhibit Bragg gaps and unusual negative refraction effects<sup>1-4</sup>. These Bragg gaps arise from destructive interference due to the periodicity of the crystals that prohibits wave propagation for a range frequencies, either in a certain direction (stop band) or in all directions (band gap). Also, in some strongly scattering media, gaps can be induced by coupling between a scattering resonance and the propagating mode of the embedding medium<sup>5,6</sup>. In the context of phononic crystals, this type of bandgap has been called a “hybridization gap”<sup>7,8</sup>.

Recently, the coexistence of a hybridization gap and a Bragg gap at different frequencies in a 3D phononic crystal has been reported<sup>7</sup>. Here, we focus on 2D crystals, where the lattice constant  $a$  can be tuned by changing the separation between the scatterers. Since the bandgap frequencies these two types of gap depend differently on  $a$ , their different physical origins can be distinguished experimentally. Further confirmation of the different character of the hybridization gaps in our crystals is revealed by comparison with a random sample, where the hybridization gap persists but the Bragg gap is destroyed.

The 2D phononic crystals consisted of 6-layers of 0.46-mm-diameter nylon rods immersed in water. The rods were made from fishing line held under tension at both ends. The rods were held in place by PVC plates in which holes were drilled in a triangular lattice configuration. Two phononic crystals with different lattice constants  $a$  and one sample with randomly arranged rods were prepared; sample details are given in **Table I**. The relevant material properties of nylon are: density  $\rho = 1.12 \text{ g/cm}^3$ , longitudinal and transverse sound speeds  $c_l = 2.6 \text{ mm}/\mu\text{s}$ ,  $c_t = 1.1 \text{ mm}/\mu\text{s}$ .

Transmission experiments to investigate the band structure were conducted in a water tank using pairs of identical transducers (Panametrics) spanning the frequency range from 0.5 to 4.0 MHz. The propagation direction was along  $\Gamma\text{M}$ , normal to the surface of the crystals. Measured dispersion

Sample name	Thickness (mm)	Volume fraction	$a$ (mm)	$b$ ( $\text{mm}^{-1}$ )
PC 20	4.70	0.216	0.980	7.40
PC 40	3.46	0.415	0.693	10.47
RS 20	4.47	0.203		

Table I. Nylon volume fraction, sample thickness, and crystal ( $a$ ) and reciprocal ( $b = 4\pi/\sqrt{3}a$ ) lattice constants. The width and height of all samples is 100.0 and 150.0 mm, respectively. PC and RS stand for phononic crystal and random sample, respectively.

curves and transmission coefficient for all samples are shown in **Fig. 1**.

The existence of hybridization gaps is most clearly seen for the random sample as kinks in the dispersion relation and dips in the transmission coefficient. These gaps are also seen in the PC samples, where they occur at similar frequencies. These results show that the origin of the hybridization gaps does not depend on the structure. The largest hybridization gap occurs near 1 MHz, as shown more clearly in **Fig. 2**. Furthermore, at this gap, the dispersion curve has a negative slope, indicating that the group velocity ( $d\omega/dk$ ) is negative in this range of frequencies. The negative values of the group velocity occur because of pulse reshaping due to the anomalous dispersion, which leads to constructive interference at the leading edge of a pulse and destructive interference at the trailing edge; thus, the peak of the transmitted pulse emerges from the sample before the peak of

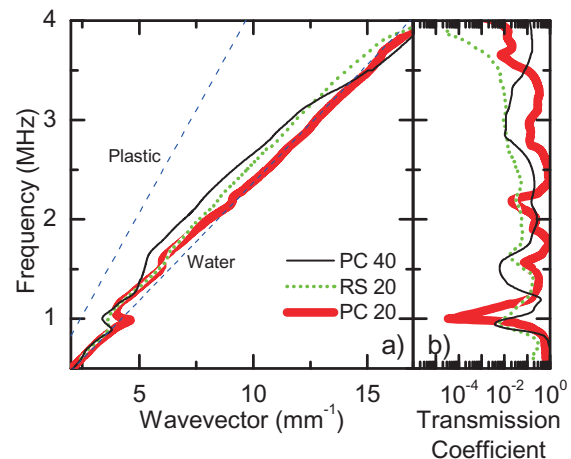


Fig. 1. a) Dispersion curves and b) transmission coefficient from 0.5 to 4.0 MHz for all samples.

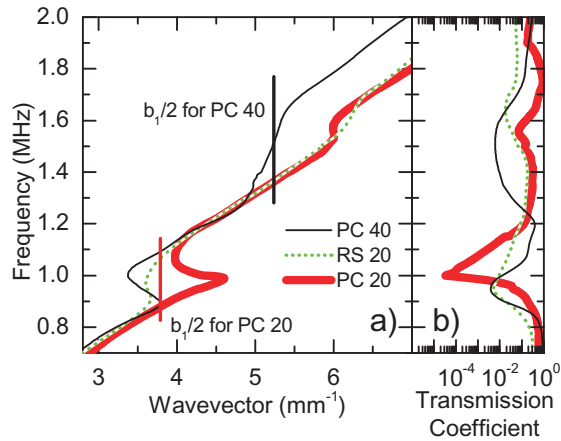


Fig. 2. a) Dispersion curves, with the edges of the first Brillouin zone marked with vertical lines, and b) transmission coefficient for frequencies near the two lowest band gaps.

the input pulse has entered it, so that the pulse transit time and hence group velocity is negative. It is noted that, at a given time, the intensity of incident wave is always greater than the transmitted one, so that the relativity is not violated. By comparison, Bragg gaps are observed only in the crystals, and have positive group velocity. This behaviour of the group velocity reveals the different nature of the hybridization and Bragg gaps.

In PC 20, the lowest hybridization and Bragg gaps near 1 MHz overlap, producing a single “joint” gap (see Fig. 2). This joint gap is shifted up in frequency relative to the hybridization gap in the random sample, and exhibits a very sharp kink in the dispersion relation and deep minimum in the transmission coefficient. This frequency shift is most apparent in the transmission coefficient. Figure 2 also shows a second hybridization gap at 1.6 MHz in both RS 20 and PC 20. This gap is well separated from the Bragg gaps in PC 20, allowing the effect of crystalline order on the shape of the hybridization gap to be observed. The hybridization gap in PC 20 is sharper than in RS 20, as the kink in the dispersion curve is more distinct and the dip in transmission coefficient is narrower. This difference reflects the broadening of the gap in the random system due to a separation-dependent coupling between resonances of nearby rods. In the phononic crystal, all neighboring rods are at the same separation so that the frequency shift associated with the coupling between resonances is the same, while in the random system, there is a wide range of separations between nearest-neighbor rods and a corresponding distribution of resonance frequencies, with a corresponding broadening of the hybridization gap.

In PC 40, the lowest hybridization gap at 1 MHz and the first Bragg gap at the Brillouin zone

boundary are well separated (see Fig. 2), allowing the anomalous dispersion associated with the lowest hybridization gap to also be measured. However, the Bragg gap in PC 40 coincides with the second hybridization gap at 1.6 MHz, producing a single very broad gap at the Brillouin zone boundary. As the second hybridization gap is weaker than the first, this joint gap appears to be dominated by Bragg scattering effects, with the shape of the dispersion relation preserving the character of a Bragg gap<sup>2</sup>.

PC 20 and RS 20 have similar volume fractions (0.216 and 0.203, respectively), so that one might expect the dispersion curves in the pass bands to be close together, in between the plastic and water dispersion lines. However, the dispersion curve of PC 20 is significantly closer to the water dispersion line above 2 MHz (Fig. 1), even though it has the slightly higher volume fraction. Also, the high frequency transmission coefficient in RS is smaller, reflecting the effect of increased scattering due to disorder.

In conclusion, we have demonstrated the coexistence of hybridization and Bragg gaps in 2D phononic crystals. By varying the lattice constant, both isolated hybridization gaps and joint hybridization-Bragg gaps have been observed. The hybridization gaps are robust in that they take place in all samples at the same frequency, while Bragg gaps are destroyed by disorder and shift in frequency as the lattice constant is varied. In the hybridization gaps, the group velocity was found to be negative, with this effect being especially pronounced in the phononic crystals compared with the random sample.

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