3P5-24

Proceedings of Symposium on Ultrasonic Electronics, Vol. 30 (2009) pp. 569-570
18-20 November, 2009

Physical Considerations on Bernoulli's Law for Mitral Valve Regurgitation

僧帽弁逆流のベルヌーイ則に関する物理的考察

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1. Introduction

The simplified Bernoulli's equation is widely used in cardiology to measure the pressure difference between the left ventricle and the left atrium. After Holen *et al*.[1)] first applied Bernoulli's equation to mitral stenosis using assumptions valid for mitral stenosis conditions, many clinical studies[2,3)] of not only stenosis but also mitral regurgitation have been reported.

Even though the simplified Bernoulli's equation is a standard method, its derivation has not been well clarified, which may lead to inappropriate usage of the equation. In addition, though there have been a few studies[4,5)] examining the accuracy of Bernoulli's equation for valve stenosis, no study regarding its accuracy for valve regurgitation has been reported. Accuracy of Bernoulli's equation for regurgitation needs to be investigated, since regurgitation flow regimes have a much larger velocity scale and smaller time scales than flows through a stenotic valve.

In the present study, the the accuracy of the Bernoulli's equation for valve regurgitation case is discussed. First, we review the derivation of the simplified Bernoulli's equation and list the assumptions used. Second, the validations of the assumptions are examined for valve regurgitations to indicate the appropriate ranges where the simplified Bernoulli's equation is valid. The numerical simulation results will be presented in the presentation.

2. Review of the Simplified Bernoulli's Equation

The derivation of the simplified Bernoulli's equation is described in this section, and we review and point out four assumptions used in the derivation process. Each assumption is indicated by "(A.)".

Starting from the full Navier-Stokes equation, which is the strictest equation of fluid motion, the unsteady Bernoulli's equation along a streamline path described in Eq. (1) is derived by assuming that the flow is incompressible (A.1) and irrotational (A.2).

$$\int_1^2 \frac{\partial V}{\partial t}\,dl + \frac{P_2}{\rho} + \frac{V_2^2}{2} + gH_2 = \frac{P_1}{\rho} + \frac{V_1^2}{2} + gH_1, \qquad (1)$$

where V, P, ρ, g, and H denote velocity, pressure, density, gravitational acceleration, and height, respectively. The subscripts correspond to positions of interest shown in **Fig. 1.** The steady state assumption (A.3) also simplifies Eq. (1).

$$P + \frac{\rho}{2}V^2 + \rho g H = const. \qquad (2)$$

When the height difference between the positions is negligible, the assumption of negligible potential energy difference, $\rho g \Delta H$, (A.4) is justified. Then, the well known simplified Bernoulli's equation[3)] described in Eq. (3) is derived by changing the pressure units from Pa to mmHg. The constant C_B in Eq. (3) is 3.975 mmHg s^2/m^2, or simply 4.0 mmHg s^2/m^2, with a blood density, ρ, of 1060 kg/m^3 and a unit-converting constant between mmHg and Pa, C_u, of 133.3 Pa/mmHg.

$$P + C_B V^2 = const. \quad \left(C_B = \frac{\rho}{2C_u} \right) \qquad (3)$$

3. An Additional Assumption Required for the Conventional Form of the Simplified Bernoulli's Eq.

The simplified Bernoulli's equation is often written in the form[4)] of Eq. (4) to indicate the pressure difference between the left ventricle and left atrium, P_{LV}-P_{LA}, using the mitral valve regurgitation velocity, V_{MV}.

$$P_{LV} - P_{LA} = C_B V_{MV}^2 \qquad (4)$$

Even though this form of Bernoulli's equation is standard, the derivation from Eq. (3) to Eq. (4) has not been well clarified, which may lead to inappropriate usage of Bernoulli's equation.

Here, we justify the derivation of Eq. (4), and specify the assumption behind it. Note that the derivation is also physically interesting, since it is a potential pseudo closure problem.

Assuming that the positions 1–3 in **Fig. 1** correspond to the left ventricle (LV), the mitral valve (MV), and the left atrium (LA), the simplified Bernoulli's equation can be described as

$$P_{LV} = P_{MV} + C_B V_{MV}^2 = P_{LA} + C_B V_{LA}^2 \qquad (5)$$

where we assume that V_{LV} is negligible.

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The question is how to derive Eq. (4) using Eq. (5). By considering mitral regurgitation, V_{MV}, to be a measured value, four unknowns (P_{LV}, P_{MV}, P_{LA}, V_{LV}) exist, though two equations (Eq. (5)) are presented. This is a kind of closure problem caused by having more unknowns than equations.

The closure problem could be solved using the additional assumption

$$P_{MV} \cong P_{LA}\,. \qquad (6)$$

This assumption is often applied to subsonic jet regimes in aero fluid dynamics. In subsonic jet flows, the pressure at exit is the same as the ambient pressure[6)] (A. 5). Thus, the relation in Eq. (6) can be assumed. As shown in **Fig. 2**, mitral regurgitation can be considered as a jet flowing into open space. In such a regime, the left atrium should be large enough to satisfy the jet condition. By substituting Eq. (6) into the center equation of Eq. (5), Eq. (4) can be derived.

4. Discussion of assumptions

We summarize each assumption listed above in **Table 1**. With representative scales for valve regurgitation of V_{MV} = 4m/s, ΔH= 4mm, g= 10 m/s^2, order estimations for each term were conducted. To examine the unsteady term, this momentum conservation law in control volume used:

$$\frac{\partial}{\partial t}\int_v \rho V dv + \int_S \rho VV dA = \Sigma F\,, \qquad (7)$$

where v and A denote volume and surface of the control volume and F denotes force acting it.

By the order estimation, the unsteady and potential terms were two orders smaller than the inertial term. To assume Eq. (6), the opening area of the valves needs to be sufficiently small. Severe regurgitation or a mild valve stenosis may violate this assumption.

More quantitative discussions with numerical simulations of how the jet flow model is similar to valve regurgitation will be shown in the presentation. The accuracy of each assumption needs to be considered for appropriate application of the simplified Bernoulli's equation.

4. Conclusions

By examining the widely used Bernoulli's law for mitral valve regurgitation, we concluded the following.

- The unsteady term and potential energy term are two orders smaller than the inertia term. The accuracy of each assumption needs to be considered for appropriate usage of the simplified Bernoulli's equation.
- By deriving Bernoulli's equation for flow through valves, we pointed out that the equation may involve a possible closure problem when the opening area of the valve is large.

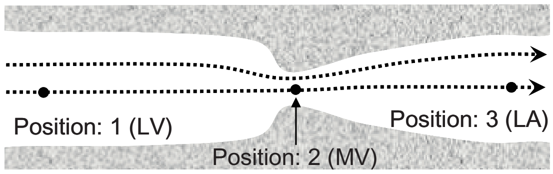

Fig. 1 Schematic of conventional Bernoulli's law. A flow goes from left to right in a tube with varying inner diameters. The dotted lines show streamlines.

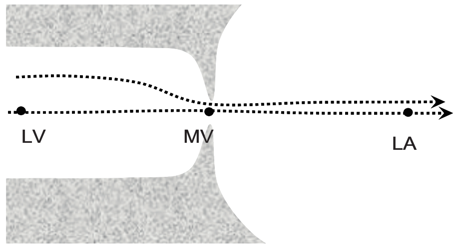

Fig. 2 Schematic of jet model of Bernoulli's law. A flow goes from left to right in a tube with varying inner diameters. The LA is open space.

Further quantitative discussions using simple numerical simulation of jet flow model similar to valve regurgitation will be presented in the presentation.

References

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Table 1 Assumption lists

Assumptions	Conditions to satisfy the assumptions
Incompr.	Satisfied for blood flows at low Reynolds number.
Irrotational	At the exit of the jet, irrotational condition can be maintained.
Steady state	Inertia term in momentum conservation of Eq. (7) is negligible. The inertia term is generally two orders smaller than the others. If the regurgitation volume is large, the assumption is not justified.
Negligible ΔH	Potential energy effect in heart is two orders smaller than the others.
P_{MV}=P_{LA}	The jet hole representative scale is much smaller than the LA diameter.