

Effects of ultrasonic multiple scattering by microbubbles on Doppler frequency spectrum

マイクロバブル群の多重散乱によるドップラ周波数分布への影響

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1. Introduction

Since intravenously injected microbubbles is capable of increasing ultrasonic echo intensity, it is effective to visualize the distribution of microvascular system in contrast echo imaging. In 2007, the Ministry of Health, Labor and Welfare in Japan approved a new contrast agent, Sonazoid, to use in diagnosis of hepatic cancer. It is important to measure slow blood flow such as in case of diagnosis of hepatocellular carcinoma, we proposed the Doppler imagings using two ultrasonic beams with different frequencies, where scattered waves of sum or difference frequency generated by nonlinear vibration of microbubbles are utilized to improving the contrast echo to tissue echo ratio^(1,2). In the Doppler method particularly using the sum frequency component generated by microbubbles, the broadening of Doppler frequency distribution was observed.

The broadening effect is possibly caused by 1) ultrasonic beam spreading, 2) velocity distribution of microbubbles (as scatterers), and 3) multiple scattering among microbubbles. This paper concentrates on the effect of multiple scattering, which may enhances the Doppler frequency broadening phenomenon. We propose the basic model and discuss the effect of multiple scattering on Doppler frequency spectrum.

2. Basic principle: Doppler frequency

In multibubble system, the acoustic wave scattered by a bubble is scattered again by another bubble. When these two bubbles has different velocities, the frequency of the scattered wave should be shifted by Doppler effect between two bubbles. This scattering phenomenon along with Doppler effect should be repeated. Assuming that the scattering is repeated M times (i.e. in the case of M-th order scattering), the Doppler frequency f_d is expressed as

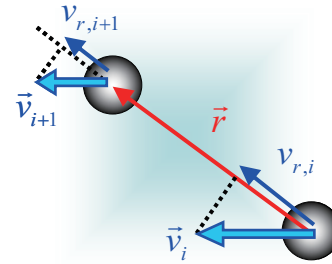


Fig. 1 Doppler effect between two bubbles.

$$f_d = f \times \frac{c + v_1 \cos \theta}{c} \prod_{i=2}^M \left(\frac{c - v_{r,i}}{c - v_{r,i-1}} \right), \quad (1)$$

the subscript i shows the value for the i-th bubble. f is the frequency of the incident sound. v_i is velocity of the first bubble and θ is the angle between the sound axis and the velocity. v_r is velocity component in the direction of vector \vec{r} as shown in Fig.1.

When bubbles moves at random (i.e. bubble velocities v_{ri} are stochastically determined by the Gaussian distribution), a frequency shift of a scattered wave slightly differs from that of the others. As a result, The frequency distribution of received wave is broadened in the Doppler spectrum.

3. Calculation and Results

In the case of static bubbles, an expression of multiple scattered wave is described in Ref. [3]. The sound pressure of the M-th order scattered wave can be obtained by additionally consideration of eq. (1),

$$P_M = p_{inc} e^{j\omega_d t} \times \prod_{i=1}^M \left(\frac{S_i}{r_i} e^{-jkr_i} \right), \quad (2)$$

$$S_i = R_{0,i} \frac{1 - \omega_{0,i}^2 / \omega_d^2 - j\delta / \omega_d}{(1 - \omega_{0,i}^2 / \omega_d^2) + \delta^2 / \omega_d^2}. \quad (3)$$

p_{inc} represents the sound pressure of the incident wave. S is the scattering function of the i-th

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bubble, where ω_0 , δ , and R_0 are natural angular frequency, damping factor and initial bubble radius, respectively. r is the distance from the i -th bubble.

To investigate how much the spectrum of received wave is broadened, we conducted the calculation. The model is shown in **Fig.2**. It was assumed that continuous incident wave with frequency of 4.8 MHz was emitted from a sound source. Bubbles were randomly located in cubic sample volume of 1 mm^3 . The number of bubble N was 300, which corresponded to the bubble concentration of 3×10^{11} . The bubble radius was distributed with the standard deviation $\sigma_r = 0.5 \text{ }\mu\text{m}$ and the average $\mu_r = 1.5 \text{ }\mu\text{m}$. In the sample volume, bubbles flowed with the velocity which was expressed as

$$\vec{v}_i = \vec{v}_{cont} + \vec{v}_{rand,i},$$

where

$$\vec{v}_{cont} = (v_{cont} \cos \theta, 0, v_{cont} \sin \theta).$$

The constants v_{cont} and θ were 0.05 m/s and 30 deg. The magnitude of $\vec{v}_{rand,i}$ was randomly determined by normal distribution with the standard deviation $\sigma_v = 0.01 \text{ m/s}$ (the average $\mu_v = 0$). The direction of $\vec{v}_{rand,i}$ was determined at random.

M -th bubbles were randomly selected in N -bubble system. By using eqs.(2) and (3), we calculated the sound pressure of scattered waves from first to M -th order. The above processes were repeated N times. M -th order scattered wave was estimated by synthesizing calculated waves. The quadrature detection was performed to analyze the Doppler shift frequency.

Figure 3 (a) shows the time profiles of the wave obtained by the quadrature detection. The solid, dashed and dotted lines show the first, fifth and tenth order scattered waves, respectively. **Figure 3 (b)** shows the Doppler frequency spectra. The solid line shows the spectrum of the first order scattered wave. Dashed and dotted lines show the spectrum of the synthesized wave from first to fifth order waves, and that from first to tenth order waves. We can see that the side lobe levels in cases of higher order (dashed and dotted lines) are larger than that in case of the first order. As a result, the spectrum seems to be broadened. On the other hand, the center frequency of main lobe almost does not change around the Doppler shift frequency of 277 Hz without consideration of velocity distribution (i.e. in the case of $v_{rand} = 0$). This result may suggest that the flow velocity can be evaluated the frequency where the amplitude is the maximum.

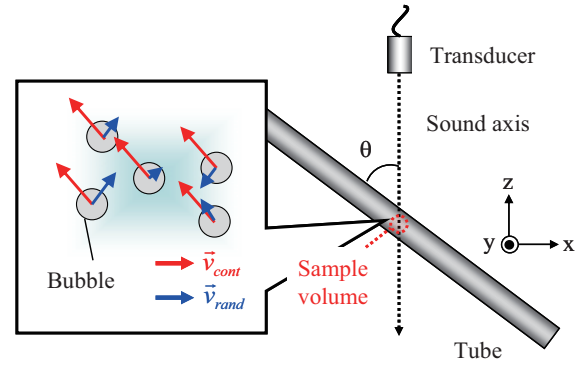


Fig. 2 Calculation model.

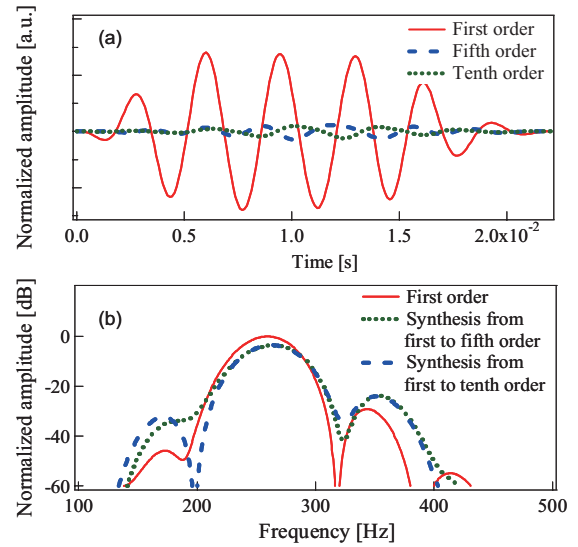


Fig. 3 Effect of multiple scattering wave on Doppler frequency spectra. (a) Time profile of the calculate wave. (b) Doppler frequency spectra.

4. Conclusions

We investigated the broadening of Doppler frequency distribution affected by multiple scattering. We considered the multiple scattering of microbubbles and derived the expression of Doppler frequency in echo signal. By using the expression, we calculated Doppler frequency spectra in N -bubble system. As a result, the spectrum seemed to be broadened by multiples scattering. However, we suppose that the flow velocity can be evaluated by the frequency where the amplitude is the maximum.

References

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