

## An investigation on ray tracing algorithms in Lamb wave tomography

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### Abstract

This paper presents an investigation for ray tracing of Lamb wave utilizing a quantitative laser ultrasound visualization technique (QLUV) and two-dimensional wavefronts model. A novel technique called Lamb wave tomography had been made for image restructuring of the defect region information of specimen, including position, size and shape. Previous researches developed existing algorithm to restructure image in Lamb wave tomography. However these restructured algorithms are susceptible to refraction, strong scattering or abrupt change in the thickness and result inaccurate image reconstruction. This paper employs QLUV as experimental ray tracing method for Lamb wave propagating along samples suffering strong refraction on beam distortions. Based on the experimental ray tracing, the existing algorithms called Vidale's method are shown for comparison with QLUV method.

**Keywords:** Laser ultrasound technology, Ray tracing, Lamb wave tomography, Quantitative Laser Ultrasound Visualization Technique

### 1. Introduction

Lamb wave inspection is an effective nondestructive evaluation (NDE) method. Because Lamb wave can propagate over long distance and the wave behavior will be influenced by material and geometry properties. In recent decade, a new inspective technology called Lamb wave tomography is developed for inspection of industry [1~3]. Almost techniques assume that Lamb wave propagating path is straight ray. However the ray straight assumption does not be established in normal Lamb wave propagating phenomenon, especial for scattering, refractive or extreme changing in the thickness. Therefore the straight ray algorithm must be corrected[4~6]. In previous studies, Lytle and Andersen[4,5] pointed out the difficult image reconstruction in different refractive index media, and Lytle simulated the ray tracing path using iterative ray tracing. Vidale had developed finite difference method to obtain the ray path. Recently, Malyarenko[6] combined iterative shooting method and simulated annealing ray tracing, and then add Monte Carlo optimum algorithm to overcome the image reconstruction in

the scattering and refractive regions.

Almost previous researches emphasized the image reconstruction in algorithm to discuss the scattering, refractive media and abrupt change in wave velocity. Few studies compare modified algorithm with experiment. In this research, a quantitative laser ultrasound visualization technique (QLUV) is used to obtain Lamb wave wavefronts. In ray tracing algorithm, finite difference method called Vidale's method will be utilized to simulate the ray tracing.

### 2. Theoretical model

In this study, ray tracing algorithm called finite difference (Vidale's method) is used to track the wavefronts. Following it will be introduced.

The propagation of Lamb waves are two-dimensional geometric rays and therefore the propagation of two-dimensional wavefronts are guided by the eikonal equation (1) of ray tracing that relates the gradient of the travel time to the velocity structure.

$$\left(\frac{\partial t}{\partial x}\right)^2 + \left(\frac{\partial t}{\partial y}\right)^2 = s(x, y)^2 \quad (1)$$

In equation (1),  $t$  is travelttime of guided wave. The co-ordinate axes are  $x$  and  $y$ , and  $s$  is the slowness (inverse of velocity). In this algorithm, two types for wavefront propagating approximations which are plane wavefront and circular wavefront in equation (2) and (3) individually are used to be formularized.

$$t_3 = t_0 + \sqrt{2(hs)^2 - (t_2 - t_1)^2} \quad (2)$$

$$t_3 = t_s + s\sqrt{(x_s + h)^2 + (y_s + h)^2} \quad (3)$$

The grid length of finite difference is  $h$ . By using equation (1) to (3) in finite difference method, the wavefront of guided waves can be obtained.

### 3. Experimental measurement

A QLUV is used to collect information for wavefronts of Lamb wave. **Fig. 1** shows the experimental configuration consisting of a pulsed laser for the generation of ultrasonic waves and a transducer for the detection of generated waves.

In order to obtain the information for ray path

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of Lamb wave, The laser pulse over the perimeter of the specimen and the transducer receive the signal in specific position. And then using a sequence of signal processings, the ray path of Lamb wave can be extracted.

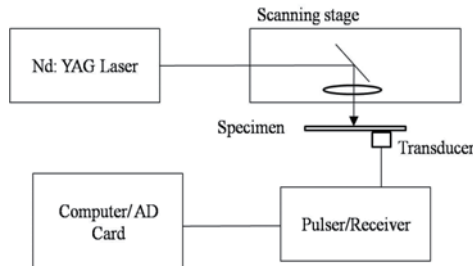


Fig. 1 A schematic showing the measurement system.

#### 4. Results and discussions

**Fig. 2** shows experimental results for the wave propagating in  $4.2 \mu\text{s}$  after laser generation. In this figure, the x, y axes represent wave propagating distance. It can be known the state of the wave transmission actually.

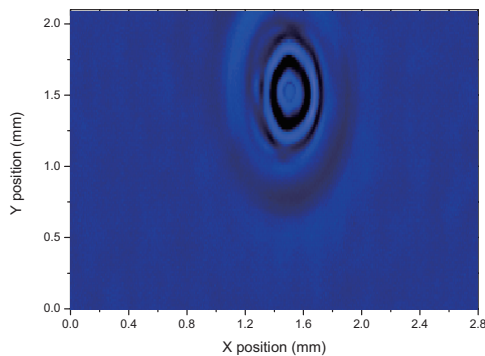


Fig. 2 Guided waves propagating phenomenon in Cu (2mm) specimen

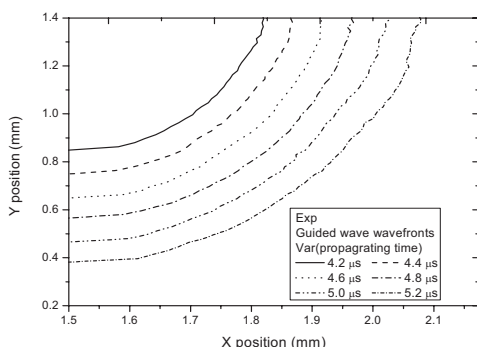


Fig. 3 Wavefronts for guided waves in different propagating time

Experimental data are utilized to analyze in the different wave propagating time. From **fig. 3**, it can be shown the wavefronts of head waves are in different time.

For **fig. 4**, it's shown the measured data of QLUV and simulated data of Vidale's method. Although there are some minor errors between experimental and theoretical data, the trends between experimental and simulated data are almost same.

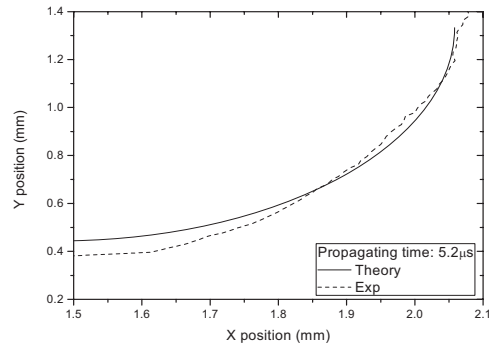


Fig. 4 Measured and simulated wavefronts for Cu (2mm) in  $5.2 \mu\text{s}$  after laser generation

#### 5. Conclusions

Finite difference for Ray tracing called Vidale's method in this research is successfully tracked and verified with QLUV. For this research, the experimental ray paths agree with the theoretical results. The next research phase, the QLUV experimental results will be provided comparisons to modify the finite difference method or other ray tracing algorithms, and offer investigations of wave propagating phenomenon like in refractive or scattering media.

Therefore the results of this research will provide a useful method and can be useful for the development of image reconstruction in Lamb wave tomography in strong refraction or other complex conditions of specimen.

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