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# Two-dimensional numerical analysis of nonlinear sound wave propagation using constrained interpolation profile (CIP) method including nonlinear effect in advection equation

移流計算に非線形効果を組み入れたCIP法による2次元非線形音波伝搬解析

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## 1. Introduction

The constrained interpolation profile (CIP) method is a highly accurate scheme in which numerical dispersion errors are hardly occured [1,2]. Using this feature, nonlinear sound wave propagation including weak shocks can be acculately calculated by the CIP method [3]-[6]. In this study, the numerical simulation of two-dimensional nonlinear sound propagation for focusing field in water is carried out by the CIP method. In the focusing field, the nonlinear effect is strongly appeared near the focal point and it makes the calculation stability decrease. To avoid this situation, the CIP method in which nonlinear effect is included in advection equation is proposed in this paper, and it is compared with the standard CIP scheme.

## 2. Implementation of nonlinear effect into CIP scheme

One-dimensional nonlinear advection equations under the assumption of weak shock are given as [3, 6]

$$\frac{\partial f_\pm}{\partial t} \pm c_0 \frac{\partial f_\pm}{\partial x} = -2\beta p \frac{\partial u_x}{\partial x} \qquad (1)$$

where $f_\pm = p \pm \rho_0 c_0 u_x$, $p$ is sound pressure, $u_x$ is particle velocity of x-direction, $c_0$ is sound speed of small amplitude, $\rho_0$ is density and $\beta$ is nonlinearity parameter. In the standard CIP scheme, eq. (1) is solved by two calculation phases; the advection phase solved by the CIP method, and the non-advection phase solved by the Euler method. In this paper, it is called as the non-advection type scheme.

Another formulation can be made based on the dependence of local sound speed on sound pressure [3, 4]. The local sound speed for the $x$-direction is given as

$$c = c_0 + \beta \frac{p}{\rho_0 c_0} \frac{|u_x|}{|u|} \qquad (2)$$

where $|u|$ is amplitude of particle velocity. The advection equations are rewritten by eq. (2) as

$$\frac{\partial f_\pm}{\partial t} \pm \left(c_0 \pm \beta \frac{p}{\rho_0 c_0} \frac{|u_x|}{|u|}\right) \frac{\partial f_\pm}{\partial x} = 0 \qquad (3)$$

The nonlinear advection can be calculated by eq. (3) as shown in **Fig.1**. In the linear case, the sound pressure represented by white circle in the figure is advected from $c_0\Delta t$ upstream point. In the nonlinear case, the sound pressures are advected from $c\Delta t$ upstream points shown in black circles. Though the sound speed (eq.(2)) must be calculated at the upstream point in the nonlinear case (black circle), the sound pressure at the upstream point in the linear case (white circle) is used instead of the nonlinear case.

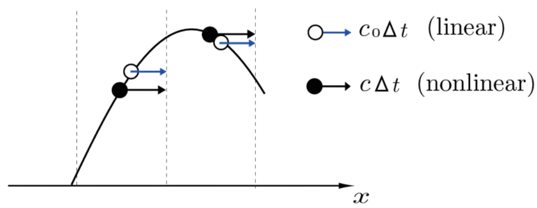


Fig.1 Advection with nonlinear effect.

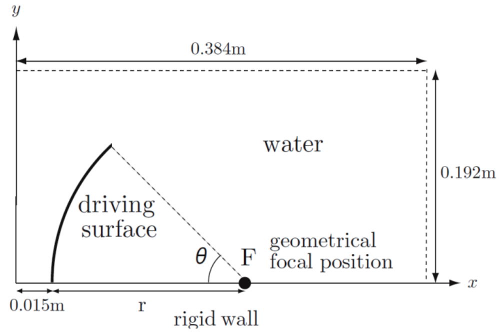

Fig.2 Calculation model.

## 4. Numerical experiments

**Figure 2** shows a two-dimensional model for the focusing sound field in water. In the model, the focal length is 0.1836 m and the angular aperture is 45°. In the numerical experiments, the grid size is chosen to be $\Delta x = \Delta y = 0.1875$ mm and the time step $\Delta t = 0.0625\,\mu$s. The sound speed $c_0$ is 1500 m/s, the medium density $\rho_0$ is 1000 kg/m$^3$ and the nonlinearity parameter $\beta$ is 3.5. The CFL number is 0.5. A single-shot pulse of sinusoidal whose amplitude and frequency are 5MPa and 100kHz, respectively, is given at around the concave surface as the initial pressure distribution at $t = 0$.

**Figure 3** shows the sound pressure waveforms of every one wavelength on sound axis around the geometrical focal position. The shock fronts are clearly calculated both in advection type scheme and the non-advection type scheme. The overshoots appear in the shock fronts due to the rapid change of the sound pressure.

**Figure 4** shows the numerical results when amplitude is increased as 10MPa. The numerical divergence occurs in the wave propagating after the focal position in the non-advection type scheme because the differential of particle velocity in eq. (1) becomes too large at the shock front. The reasonable results are obtained by the advection type scheme.


**References**
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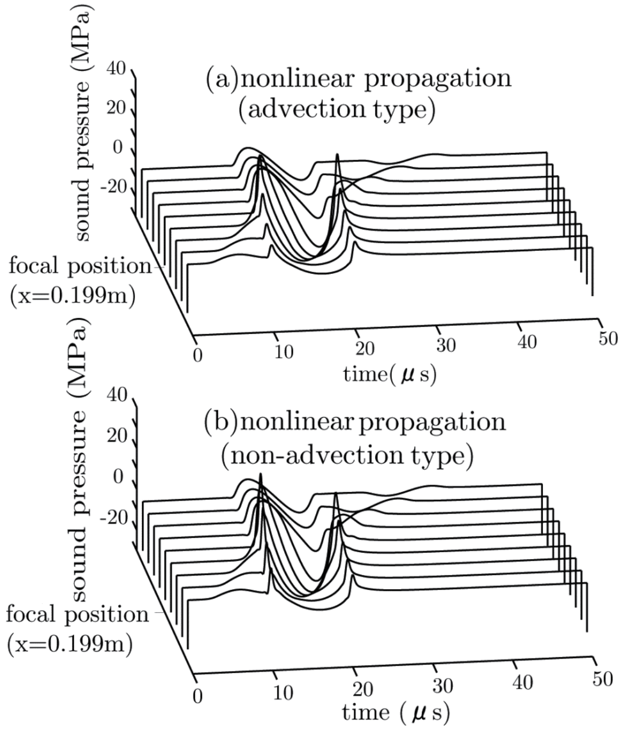

Fig.3 Nonlinear sound wave propagation. (5MPa)

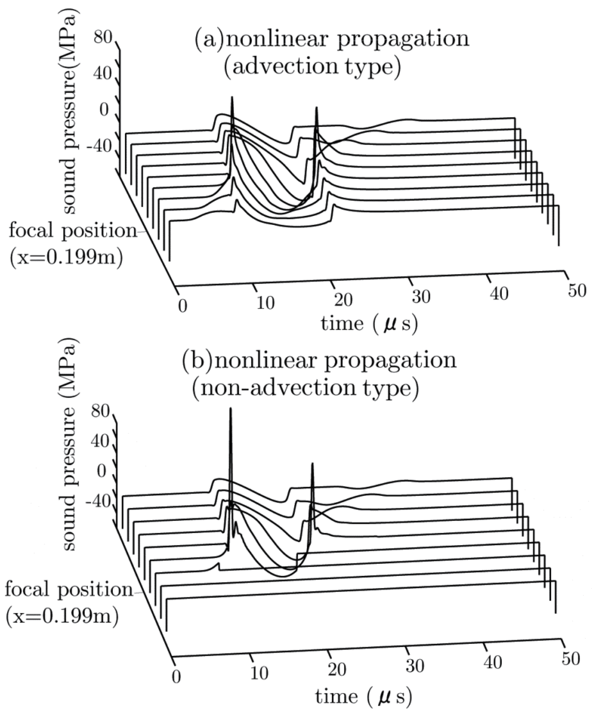

Fig.4 Nonlinear sound wave propagation. (10MPa)