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Mode Conversion of Phonons and Resonance Gap in Solid-Liquid Superlattices

固体液体超格子におけるフォノンのモード変換と共鳴周波数ギャップ

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1. Introduction

Superlattices (SLs) or phononic band-gap materials could be used for making high-quality phonon filters, phonon mirrors and vibration insulation devices in selective frequency range. In the periodic structures, the phononic band-gaps exist due to the Bragg reflections of the phonons with long wavelengths [1].

Ultrasonic band-gaps were experimentally observed in one-dimensional and two-dimensional composites for longitudinal waves [2, 3]. For example, James et al. studied the propagation of sound through a one-dimensional periodic array of water and perspex plates theoretically and experimentally [2]. The experimental result shows that the position and bandwidth can easily be engineered.

In above studies, the normal incidence case was considered. In this case, vibrational modes are decoupled from each other if the interfaces are a mirror-symmetry plane. That is, mode conversion does not occur. Only the longitudinal waves propagating through solid and liquid layers were considered. In other words, the liquid layers are used to realize the band-gap materials with broader gaps.

Recently, Hassouani *et al.* studied the sagittal acoustic waves in finite solid-liquid SL based on the Green's function method [4]. In this paper, they suggested the existence of two types of frequency gap, i.e., Bragg-type gap and the transmission zeros induced by the presence of the solid layers immersed in the liquid. However, they did not discuss the physical meaning of the transmission zeros.

In the present study, we theoretically examine the peculiar properties of phonons in solid-liquid SLs.

2. Model and Theoretical Method

1. *The isotropic continuum approximation* is used for solid layers of the SL. In this case, the phonon modes polarized in the sagittal plane are decoupled from the horizontally polarized shear mode. We consider the coupled L and T vibrations in the sagittal plane (i.e., sagittal modes).

2. *The liquid layers are assumed to be ideal*. That is, viscous shear stresses vanish in the liquid layer.

3. The normal stress S_{zz} and the normal velocity v_z should be continuous at the interfaces between solid and liquid layers, but tangential velocity v_x need not to be continuous at the interfaces.

4. Based on *the transfer matrix method*, we calculate numerically the dispersion relations of solid-liquid SLs with the infinite number of unit periods. Moreover, transmittance of phonons propagating through the finite solid-liquid SLs are calculated.

3. Numerical results and discussions

Figure 1 illustrates the phonon dispersion relations and transmittance calculated for a water/Plexiglas SL. In the phonon dispersion relations, the real part of the Bloch wave numbers (red lines) and their imaginary parts (blue lines) are shown as a function of the frequency. In the calculations of phonon transmittance, the SLs with the periods N=8 and 1 are assumed to be in water. The results for incident angles $\theta = 0°$ and 20° are shown in Fig. 1(a) and 1(b), respectively.

The thicknesses of the solid and liquid layers are assumed to be the same. Parameters we used are as follows: ρ = 1.20 g/cm³, v_t = 1.38 km/s, and v_ℓ = 2.70 km/s for Plexiglas; ρ = 1.00 g/cm³, v_ℓ = 1.49 km/s for water [4].

In the case of normal incidence, no transverse phonons can be excited. Thus, the characters of the dispersion relations are all longitudinal wave. The dispersion relations can be understood by folding the dispersion curves of a longitudinal mode into the mini-Brillouin zone determined by the period of the SL. At the zone center and boundaries, phononic band-gaps are generated. Thus, all gaps in Fig. 1(a) are Bragg-type gaps. In these gaps, the

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imaginary part of the Bloch wave number is a continuous function of the frequency, as shown in Fig. 1(a).

In the case of oblique incidence, there is another type of gap. Within this type of gap, there is a frequency at which the imaginary part of the wave number becomes infinity. We call this gap the resonance gap. Within the resonance gap, transmittance of phonons becomes zero. Even in the case of N=1, transmittance becomes zero at the resonant frequency (see Fig. 1 (b)).

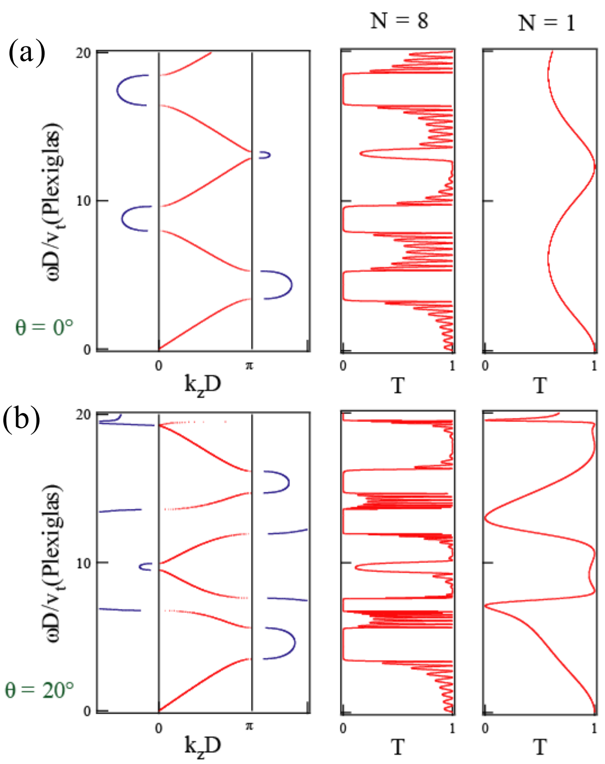

Fig. 1 The phonon dispersion relations and transmittance calculated for a water/Plexiglas SL. In the phonon dispersion relations, the real part of the Bloch wave numbers (red lines) and their imaginary parts (blue lines) are shown as a function of the frequency.

4. Phonons through a single solid layer in liquid

Even in the single layer case, there are resonant frequencies at which perfect resonant reflections occur.

In the ideal liquid layer, the transverse (T) phonons cannot be excited. When longitudinal (L) phonons are injected to an interface of liquid and solid layers at an angle from a liquid, only L phonons are reflected. As for transmitted phonons, on the other hand, both L and T phonons are involved in the solid layer. Then, the transmitted L and T phonons are scattered at the second interface, and each phonon are reflected as both L and T phonons but transmitted only as L phonons. As a result of the multiple reflections between two interfaces, perfect resonant reflections can occur, i.e., the amplitudes of the transmitted L phonons are cancelled out at resonant frequencies.

By analogy with the optics theory of the antireflection coating, we can derive a formula giving the resonant frequencies. The resonant frequencies are given by the solutions of

$$\left\{\left(\frac{k_{tz}}{k_x}\right)^2-1\right\}^2\sin\left(k_{tz}D\right)=-4\frac{k_{\ell z}}{k_x}\frac{k_{tz}}{k_x}\sin\left(k_{\ell z}D\right).$$

Here, $\mathbf{k}_{\ell z}=\left(k_x,k_{\ell z}\right)$ and $\mathbf{k}_{tz}=\left(k_x,k_{tz}\right)$ are the wave vectors of L and T phonons in the solid layer.

In the solid-liquid SLs, these resonant frequencies are gathered and generate a frequency gap (i.e., resonance gap), which is physically different from the Bragg gap.

5. Conclusions

We examined the phonon propagation in a SL consisting of alternate stacking of liquid and solid layers. There are two kinds of frequency gaps, i.e., the Bragg gap and the Resonance gap. The resonance gap is shown to be a phononic band-gap peculiar to the solid-liquid SL. The phonons in the resonance gap are multiple-reflected inside the solid layers with repeated mode conversions. These phonons are finally reflected as L phonons. This perfect reflection occurs even in the case of N=1. We derived the equation giving the resonant frequencies. In the periodic structure, these resonant frequencies are gathered and generate a resonance gap.

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