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On the perfectly matched layer for the WE-FDTD method

WE-FDTD 法における PML 境界について

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1. Introduction

The wave equation finite difference time domain (WE-FDTD) [1, 2] method is a numerical method for the analysis of sound field in time domain. In the WE-FDTD method, the wave equation is directly discretized based on the central differences. The WE-FDTD method has the same accuracy with the standard FDTD method [3], while the memory usage of the WE-FDTD method is less than the standard FDTD method because no particle velocity is stored, so the WE-FDTD method is suitable for the large-scale sound field analysis. The non-reflective boundary such as the Perfectly Matched Layer (PML) [4] is often required for the sound field analysis. The PML has been proposed for the WE-FDTD method [5], but the treatment is very complicated. In this paper, we develop the PML for the WE-FDTD method in the two-dimensional field based on the fact that the standard FDTD domain can be easily combined with the WE-FDTD domain.

2. Theory

2.1 WE-FDTD method

The wave equation for the linear two-dimensional sound field is given as

$$\frac{1}{{c_0}^2}\frac{\partial^2 p}{\partial t^2} = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}, \tag{1}$$

where p is sound pressure, c_0 is sound speed. The finite difference equation for Eq. (1) is given as

$$\begin{aligned} p^{n+1}(i,j) &= 2p^n(i,j) - p^{n-1}(i,j) \\ &\quad + \chi^2\left\{p^n(i+1,j) + p^n(i-1,j)\right. \\ &\quad \left. + p^n(i,j+1) + p^n(i,j-1) - 4p^n(i,j)\right\}, \end{aligned} \tag{2}$$

where $p^n(i, j)$ represents the sound pressure on the grid point $(x, y) = (i\Delta x, j\Delta y)$ at the time $t = n\Delta t$, and χ is the CFL number. In the sound field rendering, $\Delta x = \Delta y = \Delta$ is normally assumed because the sound field is uniform. The memory usage of the WE-FDTD method is less than 2/3 of the standard FDTD method because of no storage of particle velocity.

2.2 Combination of WE and standard FDTD methods

To combine the standard FDTD domain with the WE-FDTD domain, the particle velocity is required on the interface between two domains as shown in Fig.1. The particle velocity on the interface can be calculated from neighboring sound pressures at the grid points i and i+1 as follows.

$$\begin{aligned} \bar{u}_x^{\,n+\frac{1}{2}}(i+\tfrac{1}{2},j) &= \bar{u}_x^{\,n-\frac{1}{2}}(i+\tfrac{1}{2},j) \\ &\quad - \left\{p^n(i+1,j) - p^n(i,j)\right\}, \end{aligned} \tag{3}$$

where u_x is the particle velocity in x-direction, $\bar{u} = Z_0 u/\chi$ is the normalized particle velocity, and Z_0 is the characteristic impedance of the medium. Equation (3) is given by the standard FDTD method. For the y-direction, the same formulation can be made. Once the particle velocity is obtained on the interface, the combined domain can be calculated by the standard FDTD method.

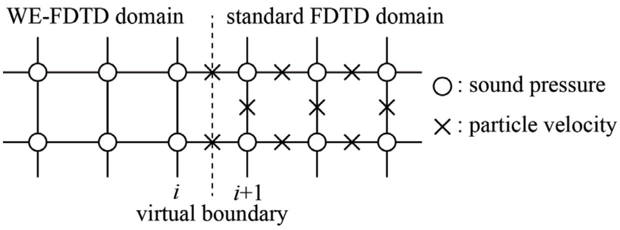

Fig.1 Interface between WE and standard FDTD.

2.3 PML

PML can be easily implemented into the WE-FDTD method by replacing the standard FDTD domain in Fig.1 by the PML domain. We here consider the numerical model in which the WE-FDTD domain is surrounded by the PML domain as shown in Fig.2. The governing equations in the x-direction in PML are given as

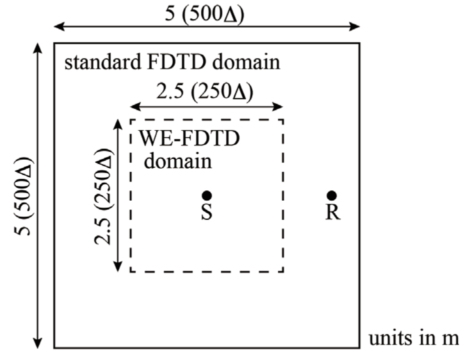

Fig.2 Numerical model.

$$\frac{\partial p_x}{\partial t} + c\frac{\partial \bar{u}_x}{\partial x} + \frac{R_x}{\rho} p_x = 0, \tag{4}$$

$$\frac{\partial \bar{u}_x}{\partial t} + c\frac{\partial p}{\partial x} + \frac{R_x}{\rho} \bar{u}_x = 0, \tag{5}$$

$$p = p_x + p_y, \tag{6}$$

where ρ is the density of the medium and R_x is the absorption constant. R_x is given as the function of distance from PML boundary.

$$R(d) = R_{\max}\left(\frac{d}{N}\right)^2, \tag{7}$$

where d is the distance from PML boundary, N is the width of PML, and $R_{\max}$ is the maximum value of R. For the y-direction, the same equations can be derived. These are the same formulations given by Berenger [4]. It is possible to implement PML easily in the WE-FDTD method without any special considerations.

4 Numerical experiments

In order to verify the accuracy of PML for the WE-FDTD method, the numerical experiments are carried out for the two-dimensional model. The medium is assumed to be air (ρ_0 = 1.2 kg/m^3, c_0 = 340 m/s), the grid size is Δ = 1.0 cm, the CFL number is χ = 0.5, and the time step is Δt = 14.7 μs.

4.1 Verification of the combination method

To verify the proposed combination method, some numerical experiments are carried out. A differential Gaussian-shaped wave of 50Δ wavelength is applied to the center of the domain as a point source as shown in Fig.2. All boundaries are assumed to be rigid. Fig.3 shows the sound pressure distribution at y = 2.5 m when t = 2.9, 4.3, 5.6 and 6.9 ms. There is no reflection at the interface between two domains, so it is confirmed that the combination method is valid.

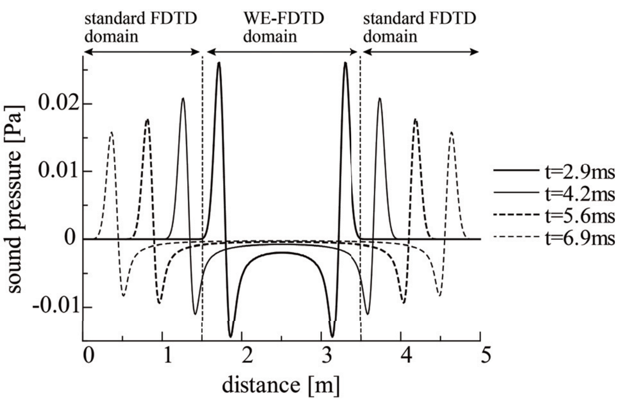

Fig.3 Sound pressure distributions at y = 2.5 m.

4.2 PML for the WE-FDTD method

Numerical experiments are then carried out for PML. Fig.4 shows the sound pressure waveform of the reflection wave from the PML surface at the observation point R=(4.5, 2.5) m. It is assumed that PML is five layered and $R_{\max}$ is 0.7. For comparison, the numerical result by the standard FDTD with PML is also shown in the figure. Two results show good agreement. Fig.5 shows the sound pressure level corresponding to Fig.4 as the incident wave is normalized as 0 dB. The non-reflective characteristic is reasonably achieved, so it is confirmed that the PML is easily introduce into the WE-FDTD method.

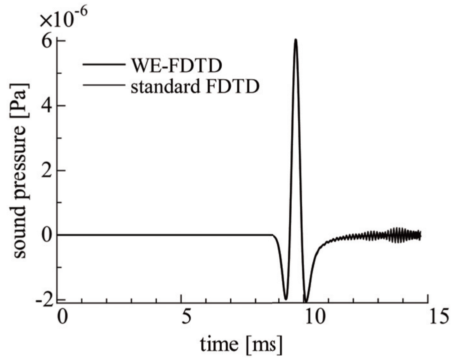

Fig.4 Sound pressure waveforms of reflection wave at the observation point R.

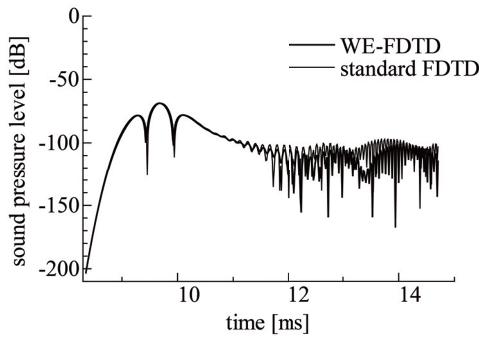

Fig.5 Sound pressure levels of reflection wave.

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