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Measurement of Liquid Viscosity and Density using Single Piezoelectric Sensor with Two Vibration Modes

二つの振動形態を有する単一圧電センサによる
液体の粘度密度計測

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1. Introduction

Recently, we have reported a technique for simultaneous measurement of liquid viscosity and density using single piezoelectric sensor. This sensor is constructed of a bimorph type resonator with two vibration modes. The technique we proposed may make measurement systems smaller and simpler than the conventional equipment[1-3)] because of using only one sensor by observing changes in two resonance frequencies of two vibration modes. In this paper, we report that the viscosity and density of a liquid are actually measured by the sensor.

2. Measurement Principle

The sensor is constructed with a piezoelectric bimorph and copper plates shown in **Fig. 1**. The edge of the bimorph is fixed and the plates are attached on the bimorph. The tip of the plate attached in *y*-*z* plane is immersed in a liquid. On each surface of the bimorph are attached two electrodes. The sensor vibrates in tangential or normal direction with respect to the contact surface by selectively driving the electrodes. To get a remarkable resonance characteristic in normal direction, extra mass is attached in *x*-*y* plane. The following equation expresses the motion equation expressing the forced vibration of the sensor in the liquid,

$$F_0 = \left[k_e - \omega^2\left(m_e + m_l\right)\right] u + j\omega\left(b_e + b_l\right) u . \qquad (1)$$

Here, F_0, k_e, m_e, b_e, ω, and u show the external force, the spring constant, the equivalent mass, the mechanical resistance, angular frequency, and the displacement of the contact surface of the sensor, respectively. Moreover, m_l and b_l are the mass loading and damping induced by liquid. When the plate vibrates in tangential or normal direction, the mass loading m_{lt} or m_{ln} are expressed by viscosity η and density ρ [1, 4, 5)],

$$m_{lt} = \frac{A}{\sqrt{2}}\sqrt{\frac{\eta\rho}{\omega}}\ ,\quad m_{ln} = 3\sqrt{2}\pi R^2\sqrt{\frac{\eta\rho}{\omega}} + \frac{2\pi R^3}{3}\rho . \qquad (2)$$

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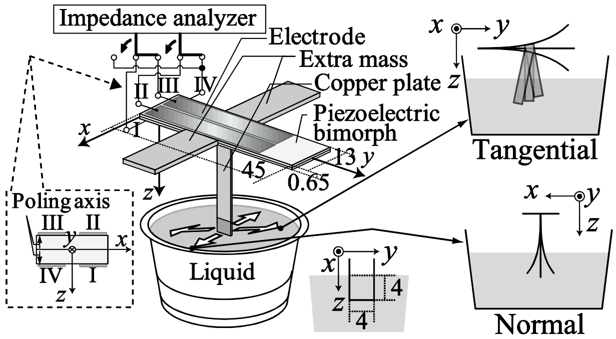

Fig. 1 Proposed sensor for measurement of the viscosity and density

Here, A and R are the area of contact surface and the radius of a sphere in the liquid, respectively. In normal direction, the vibration of the liquid around the contact surface is approximately substituted by that of the sphere. R becomes almost a half of the width of the plate, if the width and depth of the plate in the liquid are the same. In experiment, the both mass loading m_{lt} and m_{ln} can be obtained from the resonance frequencies in air f_{rAir} and liquid $f_{rLiq.}$ of the tangential and normal vibration, as follows,

$$m_{lt} = m_{et}\left[\left(\frac{f_{rtAir}}{f_{rtLiq.}}\right)^2 - 1\right],\quad m_{ln} = m_{en}\left[\left(\frac{f_{rnAir}}{f_{rnLiq.}}\right)^2 - 1\right]. \quad (3)$$

The equivalent masses m_{et} and m_{en} in each vibration mode are presumed by measuring the resonance frequencies in air and by attaching copper firm, whose mass is known, on the contact surface. Therefore, η and ρ of the liquid can be presumed by only measuring the resonance frequencies of the tangential and normal vibrations.

3. Measurements

We firstly compare m_{lt} and m_{ln} experimentally obtained to theoretical ones derived by using eq. (2) and (3) to ensure a validity of the measurement method. In this comparison, we use five standard liquids shown in **Table I**. The resonance characteristics in each vibration modes are observed by connecting sensor with an impedance analyzer as shown in Fig. 1. **Figure 2** shows the admittance amplitude and phase characteristics of each vibration mode around the resonance frequency in air and standard liquid. The resonance frequencies at

Table I Viscosity and density of standard liquid.

Liquid	1	2	3	4	5
Viscosity (mPa·s)	1.0	4.2	8.5	47	92
Density(kg/m^3)	1000	920	940	960	968

which the real part becomes the maximum are obtained from the observed admittance. The mass loadings m_{lt} and m_{ln} are derived by substituting the resonance frequencies for eq. (3). **Figure 3** shows m_{lt} and m_{ln} versus square root of product of viscosity and density. These experimental values are indicated by crosses (+), and the theoretical ones from eq. (2) by circles (○). In tangential mode shown in Fig. 3 (a), the experimental m_{lt} becomes smaller than the theoretical one. On the other hand, the experimental m_{ln} becomes larger than the theoretical one in normal mode shown in Fig. 3 (b). In the theory, it is assumed that a contact surface with infinite extent vibrates in tangential direction and a sphere vibrates in normal direction, while a rectangular plate with finite extent is actually vibrated in each direction. The difference of m_{lt} and m_{ln} between experiment and theory occurs due to rectangular edge. Thus, we devised a more generalized model. Based on eq. (2) and by introducing three independent coefficients C_1, C_2, and C_3, we have,

$$m_{lt} = C_1\sqrt{\frac{\eta\rho}{\omega}}\ , \quad m_{ln} = C_2\sqrt{\frac{\eta\rho}{\omega}} + C_3\rho\ . \qquad (4)$$

The values of the coefficients are determined by a least square method using experimental m_{lt} and m_{ln} of five standard liquids. The results using model eq. (4) are indicated by triangle (△) as shown in Fig. 3.

Next, as a practical example, the parameters η and ρ of food oil, whose value is unknown, were measured by the determined coefficients. We observe the resonance frequencies immersing the sensor in the oil, and derive the values using eqs. (3) and (4). The results are shown in **Table II**. As a reference, the values are measured by a weight meter (Shimadzu, EL-600) and a viscometer (A&D, SV-10). The density value almost corresponds to the value of weight meter while the viscosity value becomes lager than that of viscometer. The reason is considered to be that the change of resonance frequency in tangential vibration, as shown in Fig. 2(a), is so small considering the frequency resolution of the impedance analyzer. The derived product of η and ρ in tangential vibration becomes larger than true value. In normal vibration, however, the ρ is accurately derived because the body force from liquid acting as the second term of the mass loading in eq. (4) significantly decrease the resonance frequency as shown in Fig. 2(b). The larger value of viscosity η derived than actual one because the product of η and ρ has also been derived as larger value.

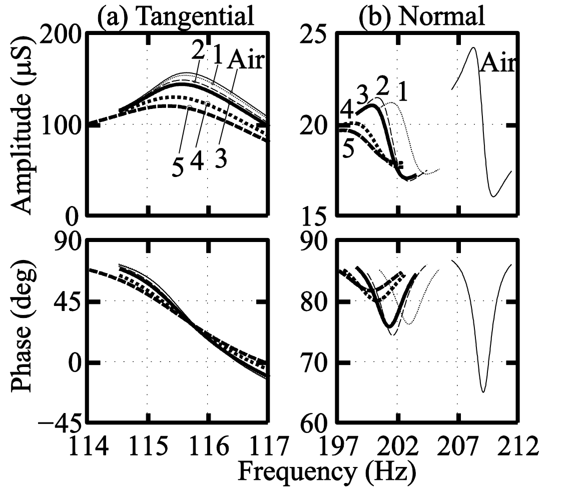

Fig. 2 Frequency response of admittance immersing the sensor in liquid: (a) tangential, (b) normal direction.

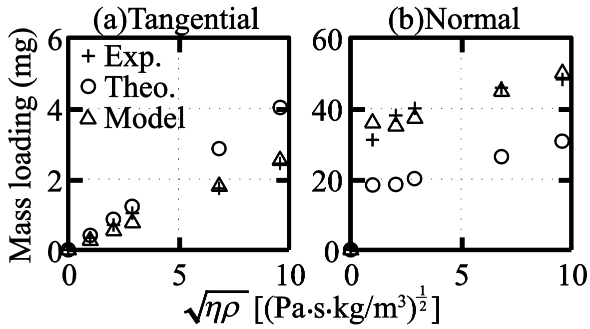

Fig. 3 Mass loading of experiment, theory, and, proposed model in standard liquid: (a) tangential, (b) normal direction.

Table II Viscosity and density of food oil derived by conventional and proposed sensor.

Property	Viscosity (mPa·s)	Density(kg/m^3)
Coventional	57.1	916
Proposed	69.2	926

4. Conclusion

We measure viscosity and density of food oil using a proposed sensor which can selectively vibrates at resonance frequency in tangential or normal direction. From the value derived by the resonance frequencies of each modes in air and food oil, the density correspond with the value of weight meter and the viscosity becomes larger than that of viscometer. The reason is considered to be that the change of resonance frequency in tangential vibration is so small considering the frequency resolution of the impedance analyzer. For the future plans, area force from liquid only in tangent direction is increased by changing the structure of the sensor.

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