

## Tunable Frequency Gaps in Piezoelectric Phononic Crystal Slabs

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### 1. Introduction

Recently, phononic crystals (PnCs) that exhibit complete frequency band gaps have attracted great interest in science and technology.<sup>1</sup> Elastic waves of any modes within the frequency gaps are forbidden. This makes PnCs being employed to obtain perfect acoustic-wave reflectors, filters, and switches. In addition, elastic energy with frequencies in the band gaps can be strongly confined to defects in PnCs, making them promising in realizing high-efficiency waveguides and acoustic wave resonators.

In many applications of PnCs, explicit control of a frequency band gap is of importance because it yields desirable operation properties. Tunable band gaps are particularly useful in controlling the elastic energy flows. Recent studies have demonstrated the tunability of frequency gaps with several means:<sup>2</sup> (i) rotation of noncircular or anisotropic inclusions to modulate the dispersion relations, (ii) mechanical deformation of the structure by an external stress, and (iii) actively changing the physical properties of the constituents of PnCs by an external field. For case (i), they are only practical for solid inclusions in gas or fluid matrix. Case (ii) is also to generate a change in geometry. The structures usually have to be made of elastomeric materials. Case (iii) is of interest due to their active control. However, only the case of bulk acoustic waves in bulk PnCs of infinite extent has been discussed up to now.

In this paper, a numerical demonstration of tunability of frequency band gaps in a PnC slab is presented. The PnC slab is composed of a square array of circular piezoelectric inclusions embedded in an epoxy matrix. Since acoustic wave velocities in a piezoelectric slab can be changed by imposing different electrical boundary conditions, dispersion relations of slab waves in piezoelectric PnC slabs can be modified. Accordingly, the frequency gaps can be tuned. A finite-element method is utilized to calculate the dispersion relations and transmission spectra to show the variation in the frequency band gaps under different electrical boundary conditions. Furthermore, by setting a configuration of electrical boundary conditions in the piezoelectric PnC slab, well guided slab wave modes to propagate along a straight path is also demonstrated.

### 2. Model and Method of Calculation

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The PnC slab is assumed to be composed of piezoelectric inclusions and isotropic matrix. The cylindrical piezoelectric inclusions with radius  $r$  are arranged in a two-dimensional square array (on the  $x$ - $y$  plane) embedded in epoxy. The lattice spacing and slab thickness are  $a$  and  $h$ , respectively. Dispersion relations are calculated with using Bloch periodic boundary conditions. Electric boundary conditions are imposed on the top and bottom surfaces of the piezo inclusions, the cases of short and open circuit electrical conditions will be discussed, respectively.

To calculate the transmission spectra, a finite PnC slab containing 5 layers of unit cells arranged along the  $x$ -direction is set up in the simulations. Single-frequency elastic plane waves are excited by a monochromatic line source to impinge on the PnC structure along the  $x$ -direction. Different polarized line sources are used. To prevent wave reflections from the domain borders, perfectly matched layers (PMLs) are implemented. By varying the excitation frequency of the line source, spectra of transmission through the finite PnC slab structure are obtained.

### 3. Numerical Results and Discussions

**Figure 1** shows the dispersion relations of the piezoelectric PnC slab under open and short circuit conditions. The assumed piezoelectric material is  $x\text{Pb}(\text{In}_{1/2}\text{Nb}_{1/2})\text{O}_3 - (1-x-y)\text{Pb}(\text{Mg}_{1/3}\text{Nb}_{2/3})\text{O}_3 - y\text{PbTiO}_3$  (PIN-PMN-PT) single crystal poled along  $[001]_c$ .<sup>3</sup> The material constants are listed in **Table I**.

In the case of open circuit, two complete gaps are found. The first gap is  $f \cdot a = 9700 - 1075$  m/s, and the second gap is  $f \cdot a = 1220 - 1370$  m/s, where  $f$  is the frequency. As the short circuit condition is imposed,

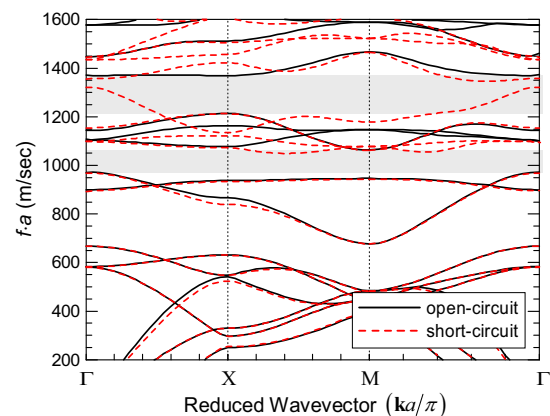


Fig. 1 Dispersion relations of the piezoelectric PnC slab under open-circuit and short-circuit conditions. ( $h=0.5a$ ).

Table I. Material constants of 0.27PIN–0.4PMN–0.33PT and Epoxy used in the simulations

Material constants	PIN–PMN–PT <sup>3</sup>	Epoxy
$c_{11}$	$10^{10}$ N/m <sup>2</sup>	12.2
$c_{12}$		11.3
$c_{13}$		10.8
$c_{33}$		11.2
$c_{44}$		6.9
$c_{66}$		6.2
$\rho$	kg/m <sup>3</sup>	8189
$e_{15}$	C/m <sup>2</sup>	16.0
$e_{31}$		-2.7
$e_{33}$		18.6
$\epsilon_{11}$	$10^{-11}$ F/m	4193
$\epsilon_{33}$		583

the frequency ranges of the band gaps are changed. Note that the second gap is changed to 1320–1360 m/s while the first gap is only reduced slightly. The significant differences suggest the tunability of the frequency gaps by changing the electrical boundary conditions. With this tunable characteristic, control of elastic wave flow can be achieved by patterning the configuration of electrical boundary conditions on the PnC slab surfaces.

**Figure 2** shows the transmission spectra with a line source of different polarizations to generate slab waves. The generated slab waves impinge on the PnC slab with five layers of unit cells along the  $\Gamma X$  direction. The transmitted power is recorded. In the figures the cases of open circuit and short circuit conditions are compared. It can be observed that the low transmission ranges can be tuned with electrical boundary conditions. The low transmission ranges are wider than the band gaps along the  $\Gamma X$  direction (Fig. 1) due to the deaf-band effect that some PnC eigenmodes can not be excited by the incident slab waves. The mixed source refers to as the  $x$ -,  $y$ -, and  $z$ -polarized sources applied simultaneously.

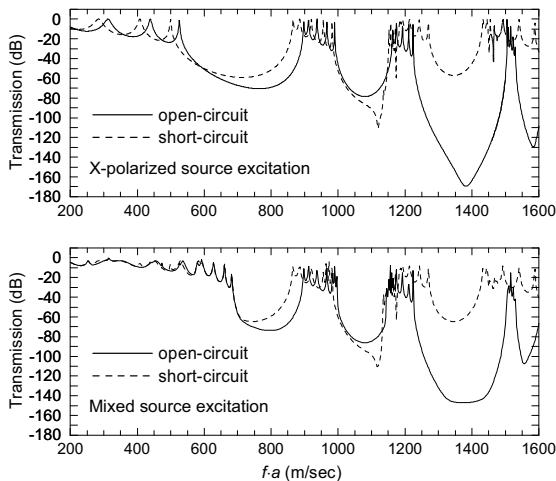


Fig. 2 Transmission through a five-layer piezo PnC slab.

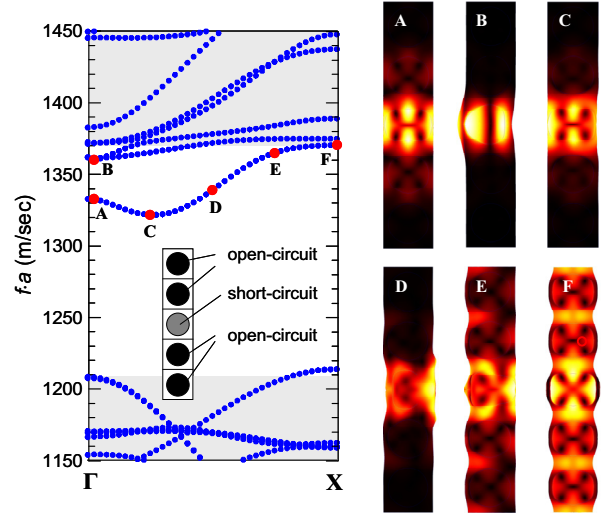


Fig. 3 Dispersion relations and total displacement fields of the piezo PnC slab with a line defect. The line defect is formed with a short-circuit boundary condition.

**Figure 3** shows the dispersion relations of the PnC slab with a line defect formed by patterning the electrical boundary conditions. A supercell method with five unit cells is utilized in the calculation. It is observed that there are frequency bands located in the second complete band gap (i.e., 1220–1370 m/s). The total displacement fields of several modes of the bands are also shown in Fig. 3. It can be seen that the modes A, B, C, and D are well confined to the electrical defect. The modes E and F are close to the gap edge so the confinement is getting poor. As a result, the waves as defect modes can be very well guided to propagate along the patterned line-defect waveguide by setting the electrical boundaries.

#### 4. Conclusions

In this study, the tunability of frequency band gaps in a piezoelectric PnC slab is demonstrated. The dispersion relations between the open and short circuit boundary conditions show variations in the band gaps. Transmission spectra also show tunable frequency gaps with the electrical conditions. With these tunable band gaps, confined propagation in a short-circuit waveguide is demonstrated.

#### Acknowledgment

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#### References

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