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Propagation of surface acoustic waves in a two-dimensional phononic-crystal layer

2 次元フォノニック結晶層における表面弾性波の伝播特性

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1. Introduction

Phononic crystals (PCs), composite materials with an elastically periodic structure, have attracted much attention for many years mainly because of the possibility of flexible manipulations of elastic waves such as modulations of band structures, frequency band gaps, guided waves and so on.

In the previous work [1] we investigated surface acoustic waves (SAWs) in two-dimensional (2D) PCs theoretically and demonstrated the existence of Rayleigh-like SAWs with folded branches and frequency gaps in the band structures. After that, many theoretical and experimental studies on SAWs in 2D PCs have been reported. [2,3]

In real experiments on SAWs in 2D PCs we have to consider the existence of the substrate covered by 2D PCs. Most of the earlier theoretical works, however, have neglected the influence of a substrate on SAWs. In this work we investigate SAWs in 2D phononic-crystal layer (PCL) attached on a semi-infinite homogeneous substrate and we elucidate the influence of the substrate on SAWs systematically.

2. Model and Methodology

The model considered here is composed of a semi-infinite homogeneous substrate (material C) and a 2D PCL coated on it. The 2D PCL with thickness h consists of a background material (B) and periodic arrays of cylinder-shaped material (A) which form a square lattice. The important parameters of the system are the combination of constituent materials (A, B and C), the thickness and the filling fraction which is defined by $f = \pi R^2 / a^2$ where R and a are the radius of a cylinder and length of each side of a unit cell, respectively. In this work we investigate the properties of SAWs in the system with these quantities systematically-changed.

A method we employed here is a plane-wave-expansion (PWE) method, which is efficient for researches on SAWs in PCs. [1,2] The equations of motion governing displacement vectors $\mathbf{u}(\mathbf{r},t)$ in the system are given by

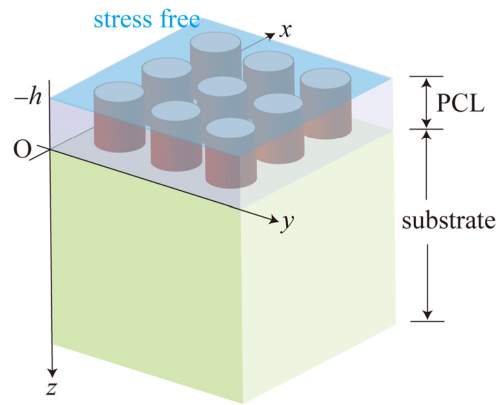

FIG1: Schematics of a 2D PCL covered on a substrate. The thickness of the PCL is assumed to be h and the stress-free boundary condition is imposed on the top surface (colored in light blue).

$$\rho(\mathbf{r})\ddot{u}_i(\mathbf{r},t) = \partial_j c_{ijkl}(\mathbf{r})\partial_k u_l(\mathbf{r},t) \quad (i = x,y,z) \quad (1)$$

where $\rho(\mathbf{r})$ and $c_{ijkl}(\mathbf{r})$ are space-dependent mass density and elastic stiffness tensor, respectively. In a PWE method, $\mathbf{u}(\mathbf{r},t)$, $\rho(\mathbf{r})$ and $c_{ijkl}(\mathbf{r})$ are expanded in Fourier series as

$$\mathbf{u}(\mathbf{r},t) = e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \sum_{\mathbf{G}} \mathbf{U}_{\mathbf{G}} e^{i\mathbf{G}\cdot\mathbf{x}} , \quad (2)$$

$$\rho(\mathbf{x}) = \sum_{\mathbf{G}} \rho_{\mathbf{G}} e^{i\mathbf{G}\cdot\mathbf{x}} , \quad (3)$$

$$c_{ijkl}(\mathbf{x}) = \sum_{\mathbf{G}} C_{\mathbf{G}}^{ijkl} e^{i\mathbf{G}\cdot\mathbf{x}} , \quad (4)$$

where $\mathbf{G} = (G_x, G_y)$ is 2D reciprocal lattice vectors and $\mathbf{x} = (x, y)$ is a 2D vector in the x-y plane. Substituting these series into Eq. (1), we can obtain a secular equation whose eigenvalues are the z-element of a wave vector $\mathbf{k} = (\mathbf{k}_{\parallel}, k_z)$ when an angular frequency ω and a wave vector $\mathbf{k}_{\parallel} = (k_x, k_y)$ in the x-y plane are given. Solving the secular equation, we obtain the general solutions for displacement vectors, which are given by

$$\mathbf{u}(\mathbf{r},t) = e^{i(k_x x + k_y y - \omega t)} \sum_{\lambda} A_{\lambda} e^{ik_z^{(\lambda)} z} \sum_{\mathbf{G}} \mathbf{U}_{\mathbf{G}}^{(\lambda)} e^{i\mathbf{G}\cdot\mathbf{x}} . \quad (5)$$

The forms of general solutions can be obtained in both the region of substrate (labelled by 'SUB') and the region of phononic-crystal layer (labelled by 'PCL').

The solutions of SAWs should satisfy the

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following boundary conditions (BCs):

$$c_{3jkl}^{(\mathrm{PCL})}(\mathbf{x})\partial_k u_l^{(\mathrm{PCL})}(\mathbf{r},t)\Big|_{z=-h}=0 \quad (j=1,2,3) \tag{6}$$

$$u_i^{(\mathrm{PCL})}(\mathbf{r},t)\Big|_{z=0}=u_i^{(\mathrm{SUB})}(\mathbf{r},t)\Big|_{z=0} \quad (i=1,2,3) \tag{7}$$

$$c_{3jkl}^{(\mathrm{PCL})}(\mathbf{x})\partial_k u_l^{(\mathrm{PCL})}(\mathbf{r},t)\Big|_{z=0}=c_{3jkl}^{(\mathrm{SUB})}(\mathbf{x})\partial_k u_l^{(\mathrm{SUB})}(\mathbf{r},t)\Big|_{z=0} \quad (j=1,2,3) \tag{8}$$

$$u_i^{(\mathrm{SUB})}(\mathbf{r},t)\Big|_{z=\infty}=0 \quad (i=1,2,3) \tag{9}$$

Equation (6) indicates $z=-h$ plane is free surface; the stresses in the z direction are zero. Equations (7) and (8) mean that the displacement vectors and the stresses in the substrate and the PCL are continuous at the $z=0$ boundary. And Equation (9) is a condition for SAWs localized around the surface, and then the displacement vectors $\mathbf{u}^{(\mathrm{SUB})}(\mathbf{r},t)$ in the substrate can be expressed by a linear combination over plane waves with a pure imaginary eigenvalue ($k_z=i\kappa$) and $\kappa>0$. A set of ω and $\mathbf{k}_\parallel$ that satisfy all the conditions gives a dispersion relation $\omega=\omega(\mathbf{k}_\parallel)$.

3. Numerical Results

The cylinder-shaped and background materials which constitute a 2D PCL are assumed to be silicon (Si) and germanium (Ge), respectively. Here, for simplicity, we approximate them as isotropic materials. In the case the velocities of transverse waves (TA) in Si and Ge are 5.4 km/s and 3.24 km/s, respectively. And in the work the filling fraction f is set as 0.3 where the velocity of transverse waves in the Si/Ge PC is 3.67 km/s within the long-wavelength limit. In the article we describe a part of our results where the substrate is made from Ge and focus on the dependence of the band structures on the thickness of PCL.

Figure 2(a) and 2(b) show the band structures of SAWs in the 2D PCL on Ge substrate for $h=0.5a$ and $h=1.7a$, respectively. The branches of SAWs obtained here are represented by filled circles. The propagation direction of SAWs is the x direction ($k_y=0$). For reference, the lowest SAW's branches in bulk Ge and 2D PCs composed of Si cylinders and Ge background material (hereafter expressed by 'Si/Ge PCs') are shown by black and blue solid lines, respectively. A branch of TA in bulk Ge is indicated by dashed line. We found that at both the thicknesses the branch of SAWs exits between the branches of the SAW and TA in bulk Ge. In addition, with large h, the branch of the SAW shifts from the one of SAW in Ge to the one of SAW in Si/Ge PC. To make the effects of the thicknesses of PCLs clearer, at each $h=0.1a$, $0.5a$, a, $1.5a$ or $2a$, the dependence of the phase velocities $c=\omega/k_x$ of SAWs in the PCL with the Ge substrate on h is shown in Figure 2(c).

At $h>1.5a$, the intersection of the SAW's branch with the TA's branch gives rise to the disappearance of the SAWs.

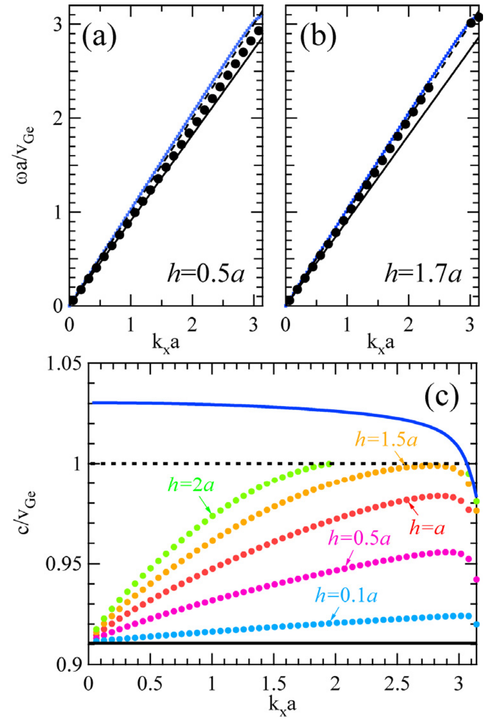

FIG2: Band structures of SAWs in a 2D PCL on Ge substrate for (a) $h=0.5a$ and (b) $h=1.7a$. (c) Dependence of the phase velocities of SAWs in the PCL with a Ge substrate on h. Black and blue solid lines indicate the phase velocities of SAWs in bulk Ge and Si/Ge PCs, respectively. Dashed line represents the velocity (v_{Ge}) of TA in bulk Ge.

4. Concluding Remarks

We have systematically investigated the propagation characteristics of surface acoustic waves in a 2D phononic-crystal layer on a semi-infinite homogeneous substrate. In the case of Ge substrate, the variation of thickness of a PCL modulates the phase velocity of the SAWs. Furthermore, with the thickness large, the intersection of the SAW's branch with the TA's branch gives rise to the disappearance of the SAWs. On the other hand, in the case of Si substrate, we found that there exist not only Rayleigh-like SAWs but also Love-like SAWs in the system, depending on the thickness of PCL and constituent materials.

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