

## Direct Measurement of Bekki-Nozaki (BN) Amplitude Holes in Nonlinear Waves on Cardiac Interventricular Septum (IVS) Wall

心室中隔壁上の非線形波動における Bekki-Nozaki ホールの直接測定

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### 1. Introduction

Human heart is the organ responsible for pumping blood through the blood vessels by repeated, rhythmic contractions. The ability to noninvasively detect regional dynamic myocardial damage related to action potentials and mechanical properties affected by heart diseases is of great clinical importance [1]. We performed the non-invasive direct measurements of Bekki-Nozaki (BN) amplitude holes in nonlinear waves on cardiac interventricular septal (IVS) wall for the healthy young male, by using the data obtained from the recent measurements of the novel ultrasonic measurement technique for myocardial motions *in vivo* found by Kanai *et al* [1].

The BN amplitude hole, has become now well known major milestones in nonlinear dynamics for complex Ginzburg-Landau Equations (CGLE) [1]. System undergoes the onset point, this status and nature of the amplitude equations which can be derived in the vicinity of symmetry breaking instabilities has been well established [1]. In this presentation, we show the validity of the experimental evidence of propagating BN hole defects of which the large scale modulation of the waves due to the IVS wall are governed by a one-dimensional CGLE [1].

### 2. Tool and Concept

In order to confirm the experimental evidence of our BN traveling hole profiles, we have developed the vector field phase gradient mapping and imaging tool. For the quantitative analysis of complex self excitatory wave pattern, such as

cardiac fibrillation requires the development of new tools for identifying and tracking the most important features of the activation, such as phase singularities. To start with, let us introduce the new tool for our purpose, the definition of the phase gradient mapping which we will be using in this study is introduced. The spatial modulation of excitable IVS can be characterized the phase  $\phi(\mathbf{r},t)$  and the local wave vector  $\nabla\phi(\mathbf{r},t)$ . In our conjecture, it is natural to assume that  $\Psi(\mathbf{r},t)$  will be able to describe in terms of  $\phi(\mathbf{r},t)$  and  $\nabla\phi(\mathbf{r},t)$  as

$$\Psi(\mathbf{r},t) = f(\phi(\mathbf{r},t), \nabla\phi(\mathbf{r},t)),$$

where  $f$  is a periodic function and we can also define the phase gradient  $\nabla\phi(\mathbf{r},t)$  rather extensively wide sense  $\nabla\phi \equiv \mathbf{k}$ .

Here, we also briefly introduce one of the typical quantitative comparison of localized amplitude holes observed in experiment with BN hole solution of the CGLE as

$$i \frac{\partial}{\partial t} \psi(x, t) + p \frac{\partial^2}{\partial x^2} \psi(x, t) + q |\psi(x, t)|^2 \psi(x, t) = i\gamma \psi(x, t). \quad (1)$$

were,  $p = p_r + ip_i$  ( $p_i < 0$ ),  $q = q_r + iq_i$  ( $q_i > 0$ ) are complex constant and  $\gamma$  ( $\gamma > 0$ ) is a real constant and  $\psi$  is a complex function of scaled space  $x$  and time  $t$ . Bekki-Nozaki discovered one-parameter family of propagating analytic exact hole solutions to the CGLE that described as “propagating holes” by means of Hirota’s bilinear operator as

$$\Psi(x, t) = \frac{b_1 e^{\kappa \xi} + b_2 e^{-\kappa \xi}}{(e^{\kappa \xi} + e^{-\kappa \xi})^{1+i\alpha}} \times \exp(Kx - \Omega t), \quad (2)$$

where,  $\xi = x - c_h t$ . The complex quantities  $b_1$  and  $b_2$ , as well as the real constant  $\kappa$ ,  $K$ ,  $\Omega$  and  $\alpha$ , depend on the parameters  $p_r$ ,  $p_i$ ,  $q_r$ , and  $q_i$  that appear in the CGLE (3). Now, a spatio-temporal hole plot of the amplitude and phase with both for the case of the late stage of systole are shown in FIG. 2 and FIG. 3. Each figure connects two waves with different wave vectors  $k_1$  and  $k_2$  for the case of spatiotemporal propagating hole defects. Propagating holes are not stationary in general, but travel at speed  $C_h$  given by

$$C_h = (p_r q_i - p_i q_r)(k_1 + k_2)/q_i \quad (3)$$

where,  $k_1$ , and  $k_2$  are the wave numbers which propagating plane waves interact with the opposite direction each other as a plane wave collision of back and forth. Figure. 1. Shows a typical example of a plane wave collision of back and forth for this situation. It is very clear understanding that in Fig. 1. the slope of  $k_1$  is positive on the contrary it's slope of  $k_2$  is negative beside a plane wave collision of back and forth.

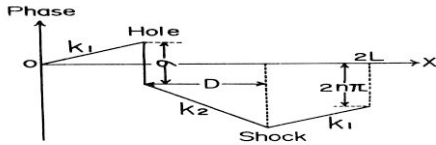


Fig. 1. A schematic figure of the phase-matching between a hole and a wave number shock in a periodic boundary condition with a period  $2L$  [ see text ] [1].

A phase-matching condition for a hole and a shock in the periodic region with a length  $2L$  is easily obtained as  $k_1(2L-D) + k_2D = 2n\pi - \sigma$ , where  $n$  is an integer,  $D$  is the distance between a hole and shock and  $\sigma$  is the phase-jump at the hole. Finally, as a result, we can confirm the strong evidence of traveling dip (Fig. 2.) and from the phase profile of Fig. 3 we could perform the direct measurements of BN hole profile.

### 3. Conclusions

We have demonstrated the experimental evidence of propagating amplitude holes defects in nonlinear waves on cardiac IVS wall, for the first time, which are successfully applied by the Bekki-Nozaki hole

solution of the CGLE. In order to find experimental evidence of BN holes, we have developed the vector field phase gradient mapping and imaging tools. We find these methods useful for analysis of complex wave trains such as chaotic cardiac excitation.

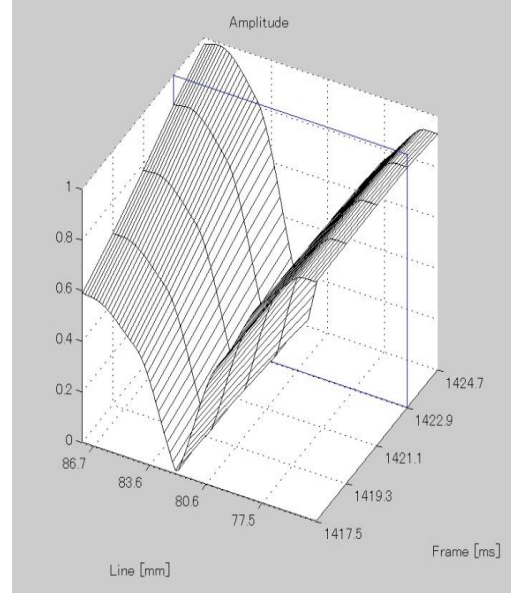


Fig. 2. Direct measurement of the Spatiotemporal localized amplitude holes plot observed in the healthy IVS wall .

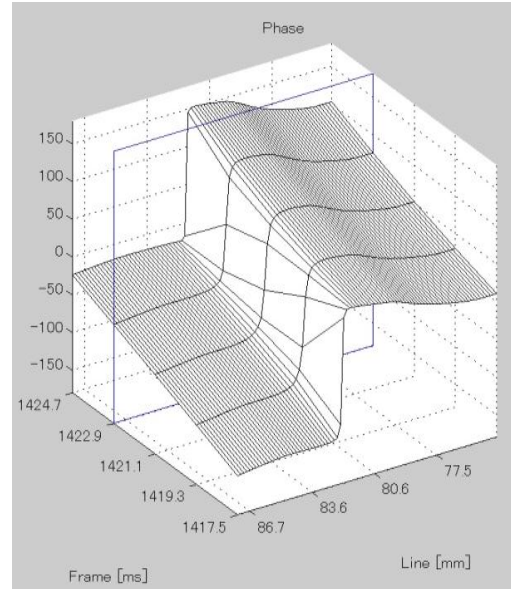


Fig. 3. Direct measurement of the Spatiotemporal localized phase plot observed in the healthy IVS wall .

### 4. References

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