

## Some Issues for Precise Lumped-Parameter Estimation in Electromechanical Coupling Systems

電気機械結合系集中定数パラメータの精密な評価における幾つかの課題

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### 1. Introduction

The lumped-parameter equivalent circuit for an electromechanical coupling transducer shown in Fig. 1 is useful from the viewpoint of modal analysis, in which the capacitance ratio  $C/C_0$  is an important factor to determine the electromechanical energy conversion, where  $C$  is a total of  $C_n$ .

The author has developed the precise estimation method for  $C/C_0$  as well as the other parameters,  $L_n$  and  $C_n$ , using the "principle of least variance on  $C/C_0$ " with regard to multiple modes, which enables us to estimate more precise electromechanical coupling coefficient than the conventional methods.<sup>1,2)</sup>

However, there are some issues to be considered in this method. For example, only one-dimensional vibration mode is considered in

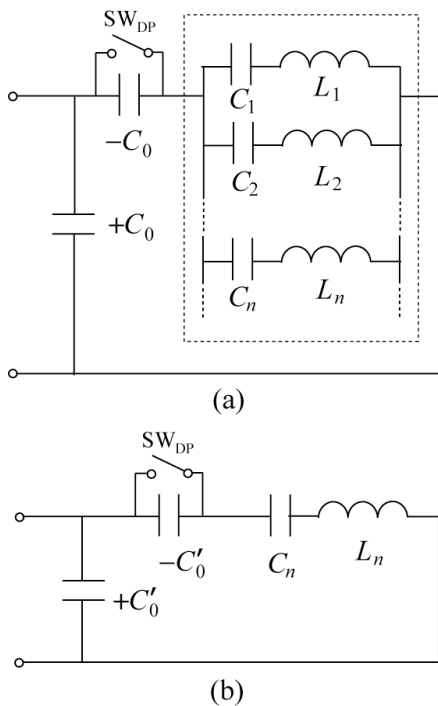


Fig. 1 (a) Equivalent circuit of electromechanical coupling transducer based on the concept of modal analysis for one-dimensional vibration. (b) Equivalent circuit for the  $n$ th mode.

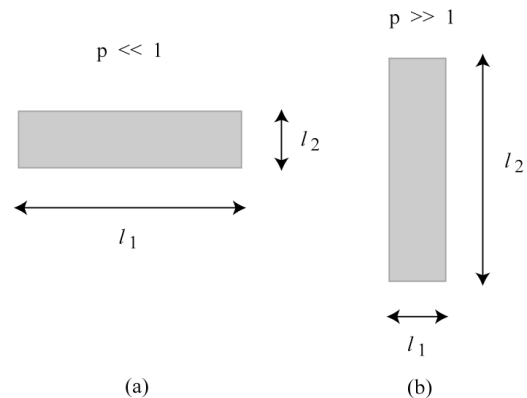


Fig. 2 Shape of transducer considering mechanical mode coupling.  $p$  is a shape factor, which is defined as the ratio of resonance frequency in the  $l_1$ -direction to that in the  $l_2$ -direction. Example of (a)  $p \ll 1$ , (b)  $p \gg 1$ .

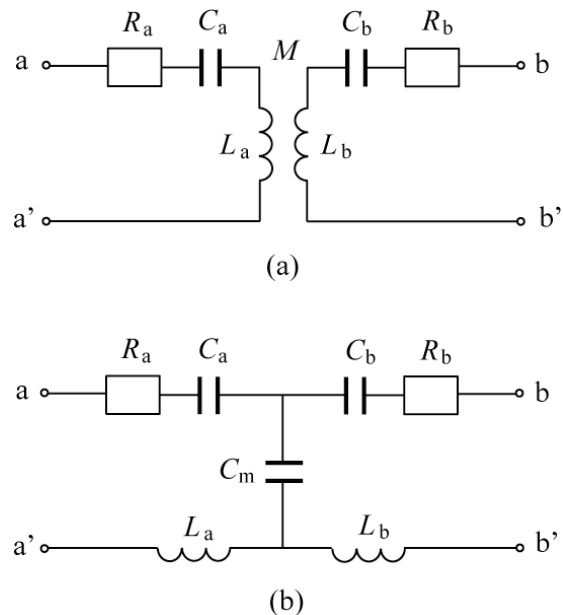


Fig. 3 (a) Inductively-coupled circuit between mode A and mode B. The mode A is driven by electric source connected at terminal a-a' and influenced by mode B (shorted at terminal b-b') via mutual inductance  $M$  due to mechanical coupling. (b) Corresponding capacitively-coupled circuit via capacitance  $C_m$ .<sup>3)</sup>

the above-mentioned circuit and methods, and the effect of mechanical mode coupling should be considered. **Figure 2** shows an example of the shape of transducer with a shape factor  $p$ . The treatment of such a coupling phenomenon can be performed using the coupled lumped-parameter circuit shown in **Fig. 3**<sup>3)</sup>, while the treatment in a distributed-parameter basis is also possible.<sup>4)</sup> In addition, some inference process is necessary for the application of the principle of least variance, relying on the mathematical nature of the resonance, as shown in **Fig. 4**<sup>5,6)</sup>, but some inference errors might be included. Due to limitations of space, this report only shows a semi-quantitative result of the influence of the mechanical mode coupling on the estimation of  $C/C_0$  using the inductively-coupled circuit shown in Fig. 3(a).

## 2. Discussion

The resonance frequency ( $\omega_n=1/(L_nC_n)^{1/2}$ ) and intensity (proportional to  $1/L_n$ ) are modified by the mechanical mode coupling, and the resonance pattern shown in Fig. 4 is changed. Therefore, the value of  $C/C_0$  obtained in the method described in refs. 1 and 2 is also changed. The value of  $C/C_0$  with the coupling is observed smaller than the case without the coupling, which is resulted from the following discussion. In the circuit shown in Fig. 3(a), the frequency dependence of the charge  $q_a = i_a/j\omega$  on  $C_a$  and the charge  $q_b = i_b/j\omega$  on  $C_b$  (corresponding to mechanical displacement) has resonant peaks modified by the mechanical mode coupling. By dividing the respective peak values by the corresponding  $Q$ -values (obtained from half width of the peaks), we can estimate the "effective" capacitance after being influenced by the

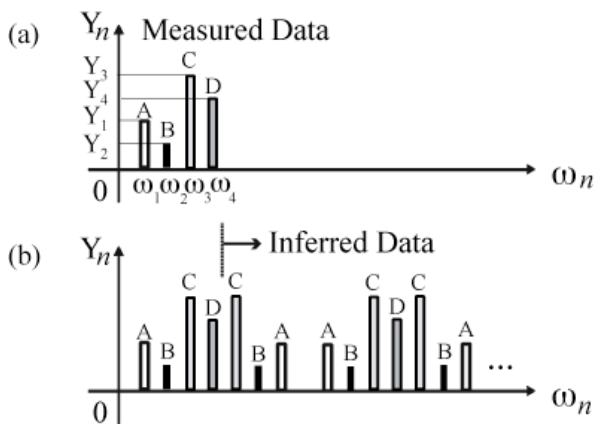


Fig. 4 Resonance intensities and resonance frequency with some pattern on the frequency domain.

mechanical mode coupling, termed  $C_a'$  and  $C_b'$ , respectively. (Remember  $C = 1/(\omega RQ)$ , in general.)

**Figure 5** shows an example of the dependence of  $C_a'/C_a$  and  $C_b'/C_b$  on the value of  $p$ . We obtain  $C_a' \sim C_a$  when  $p \ll 1$  or  $p \gg 1$  (small coupling), but  $C_a' < C_a$  for  $p \sim 1$  (large coupling), which means that the capacitance can be estimated smaller when the coupling effect cannot be neglected. (This discussion was originally performed in ref. 7 from another point of view, in which the piezoelectric properties were measured using an optical probing method, and piezoelectric characteristics were always measured smaller when  $p \sim 1$  compared with the cases of  $p \gg 1$  and  $p \ll 1$ .)

The "de-coupling" process is necessary numerically when the "true" value of  $C/C_0$  without coupling is desired. In addition, the mutual inductance  $M$  includes Poisson's ratio of the material, but the measurement of Poisson's ratio is not easy from the conventional method. We point out that Poisson's ratio can be estimated precisely, by using the optical probing method for a disc-type transducer.<sup>8)</sup>

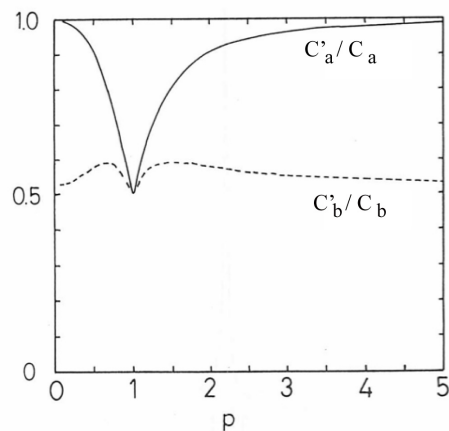


Fig. 5 Value of  $p$  versus  $C_a'/C_a$  and  $C_b'/C_b$ .

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