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Numerical analysis of sound wave propagation in ocean by WE-FDTD method with GPU cluster system

GPU クラスタを用いた WE-FDTD 法による海洋音波伝搬

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1. Introduction

For the numerical analysis of the sound wave propagation in ocean, the approximation method, such as the ray tracing method [1] or the PE method [2], is usually used because of the long distance propagation. On the other hand, the finite difference time domain (FDTD) method [3] has been applied to the analysis of the sound wave propagation in the shallow sea [4], based on the recent progress of the computational environment. However, for the analysis of the sound wave propagation in the deep sea, the FDTD method is hardly used because of the enormous consumption of the computational resources. In this paper, a GPU cluster system is applied to the numerical analysis of the sound wave propagation in ocean based on the wave equation finite difference time domain (WE-FDTD) [5, 6] method. In the WE-FDTD method, the wave equation is directly discretized based on the central differences. The WE-FDTD method has the same accuracy with the standard FDTD method [3], while the memory usage of the WE-FDTD method is less than the standard FDTD method because no particle velocity is stored in the WE-FDTD method, so the WE-FDTD method is suitable for the large-scale sound field analysis. Numerical demonstrations are made for the long distance sound wave propagation of mid-latitude areas of the Pacific Ocean assuming the sound speed profile proposed by Munk [7].

2. Theory

2.1 WE-FDTD method

The wave equation for the linear two-dimensional sound field is given as

$$\frac{1}{{c_0}^2}\frac{\partial^2 p}{\partial t^2} = \frac{\partial^2 p}{\partial r^2} + \frac{\partial^2 p}{\partial z^2}, \tag{1}$$

where p is sound pressure, c_0 is sound speed. The finite difference equation for Eq. (1) is given as

$$\begin{aligned} p^{n+1}(i,j) &= 2p^n(i,j) - p^{n-1}(i,j) \\ &\quad + \chi^2\big\{p^n(i+1,j) + p^n(i-1,j) \\ &\quad + p^n(i,j+1) + p^n(i,j-1) - 4p^n(i,j)\big\}, \end{aligned} \tag{2}$$

where $p^n(i, j)$ represents the sound pressure on the grid point $(r, z) = (i\Delta r, j\Delta z)$ at the time $t = n\Delta t$, and χ is the CFL number. In the numerical analysis, $\Delta r = \Delta z = \Delta$ is assumed because the sound field is uniform. The memory usage of the WE-FDTD method is less than 2/3 of the standard FDTD method because of no storage of particle velocity.

In the case of the sound wave propagation in ocean, the sound speed profile in z (depth) direction must be considered. The variation of the sound speed can be included in the CFL number as

$$\chi = \frac{c(z)\Delta t}{\Delta}, \tag{3}$$

where $c(z)$ is the sound speed at the depth z. χ should not exceed $1/\sqrt{2}$ which is the upper limit of the two-dimensional sound field.

3 Numerical experiments

Figure 1 shows the numerical model. The analyzing domain is assumed to be 1056.8 km × 4992 m in which the grid separation is Δ=1.5 m (20 points per wavelength), so the domain is divided into 704512 × 3328 cells. The reflection coefficient R for the sea surface is assumed to be -1, and for the sea bottom R=0.259 in which sedimentary layer (c_0=1700 m/s, ρ_0=1500 kg/m^3) is assumed. Other boundaries are assumed to be non-reflective (R=0) which corresponds to the Mur's absorbing boundary. An envelope pulse is radiated from a point source located at (0, 1000) m in which the waveform is expressed as follows

$$s(t) = \sin[2\pi f(t-t_0)]\exp\{-[w(t-t_0)]^2\}, \tag{4}$$

where f is the center frequency of the pulse, w is bandwidth. In this calculation, f=50 Hz, w=10 Hz, and t_0=0.3 s are assumed. The observation points are located at r=50 km (P1), r=500 km (P2) and r=1000 km (P3). The Munk's sound speed profile is expressed as

$$\begin{aligned} c(z) &= c_0\{1+\varepsilon(\eta-1+\exp(-\eta))\} \\ \eta &= 2(z-z_0)/B \end{aligned} \tag{5}$$

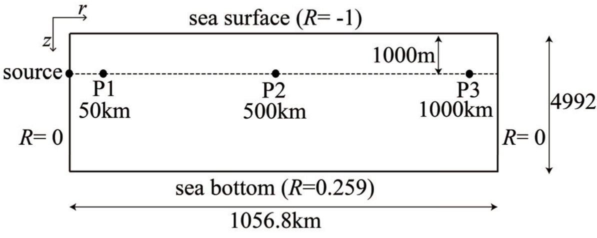
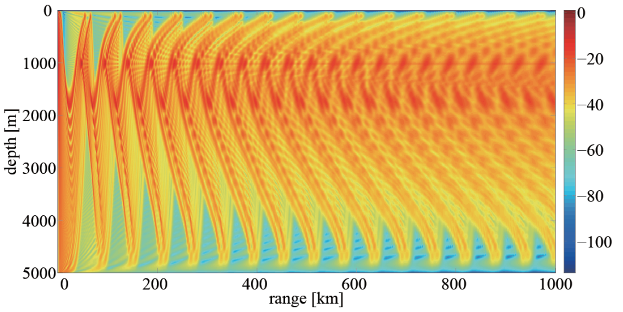
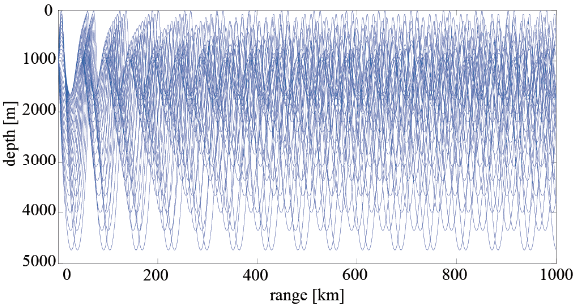


Fig. 1 Numerical model for sound wave propagation in ocean.



(a) sound intensity distribution



(b) ray tracing solution

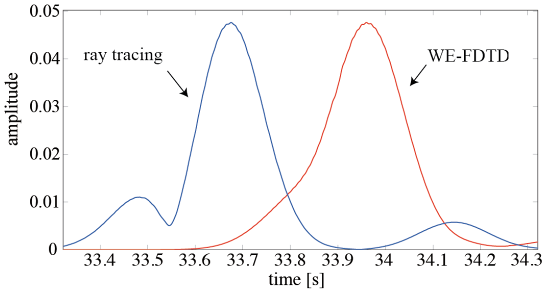
Fig. 2 Two-dimensional distribution of the sound intensity and corresponding ray tracing result.

where $B=z_0=1300$ m, $c_0=1500$ m/s, and $\varepsilon=0.00737$ in this calculation.

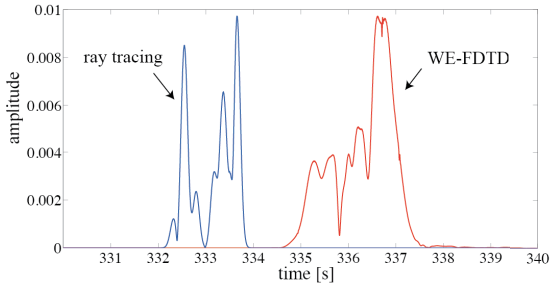
In this paper, the calculations are carried out on a GPU cluster system that consists of 32 GPUs, so the domain should be divided into 32 domains along r-direction.

Figure 2 (a) shows the sound intensity distribution calculated by the WE-FDTD method. The sound wave propagates long distance along the SOFAR channel. Figure 2 (b) shows the result calculated by the ray tracing method. Two results show good agreement in outline, so it is confirmed that the WE-FDTD method can be applied to the numerical analysis of the long distance sound wave propagation in reasonable accuracy.

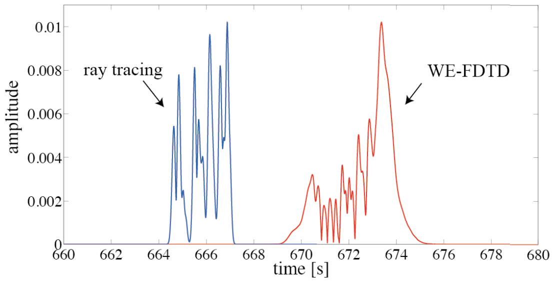
Figure 3 shows the sound pressure waveforms represented by the envelope at the observation points P1, P2, and P3. The results calculated by the ray tracing method are also shown in the figures for comparison. In the ray tracing solutions, the calculated impulse responses are convolved with the source waveform. Although two results show good agreement near the source, some differences appear in the long propagation distance because the numerical dispersion error appears in the WE-FDTD solutions. The propagation times in the WE-FDTD solutions are calculated as 0.9% delayed from the ray tracing solutions. This is again responsible for the numerical dispersion error. The calculation time is 27 hours on the GPU cluster system. It is found that at least 20 points per wavelength is required for the accurate analysis.



(a) 50km (P1)



(b) 500km (P2)



(c) 1000km (P3)

Fig. 3 Sound pressure waveforms at the observation points represented by envelope.

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