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Synthetic Aperture Ultrasound Imaging using Pseudo-inversion Operator
擬似逆演算による合成開口超音波画像の形成

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1. Introduction

A ultrasound imaging system combined from synthetic aperture imaging technique[1] and coded-excitation are shown that be possible to image in a three-dimension and high frame rate by a two-dimensional sparse transducer array.

However, because there is the relationship of the trade-off between the image quality and the frame rate, the image quality decrease when reduce number of times of transmitting and receiving for high-speed imaging.

Ebbini et al. have suggested that the method of imaging by the pseudo-inverse matrix that used the waveform of the time domain of the echo has regarded as a vector data[2].

In this report, we present the method to act pseudo-inverse matrix to a vector data of spatial dimension that be determined by number of transmitters and receivers for each frequency component[3]. We investigated the association with the method of regularization in pseudo-inverse matrix calculation, number of singular values to be used, and number of receivers.

2. Imaging by pseudo-inverse matrix

A ultrasound is transmitted from all transmitters at the same time by using sine wave signal that has been modulated phase in Walsh function[4]. When using N receivers and the field of view has discretized in the direction of L, the transfer matrix that used the complex amplitude of the echo at frequency f that detected in the i-th receiver is constructed as follows:

$$H = \begin{pmatrix} H_1^{(1)}(f) & H_1^{(2)}(f) & \cdots & H_1^{(L)}(f) \\ H_2^{(1)}(f) & H_2^{(2)}(f) & \cdots & H_2^{(L)}(f) \\ \vdots & \vdots & & \\ H_N^{(1)}(f) & H_N^{(2)}(f) & \cdots & H_N^{(L)}(f) \end{pmatrix}. \quad (1)$$

↑ The echo from the direction of 1st. ↑ The echo from the direction of 2nd. ↑ The echo from the direction of L-th.

This matrix is the size of $N\times L$. Then, pseudo-inverse matrix have been solved as follows:

$$H^+ = \sum_{k=1}^{r} \frac{1}{\beta + \lambda_k^{1/2}} u_k v_k^* , \quad (2)$$

where u_k is an eigenvector of HH^*, v_k is an eigenvector of H^*H, and $\lambda_k^{1/2}$ is a singular value of the transfer matrix H. β is the regularization parameter that has been introduced in order to suppress the amplification of unnecessary noise.

When to act a vector data consisting of the complex amplitude of the received waveform to this pseudo-inverse matrix, the following values obtain at frequency f.

$$\Phi(f) = H^+ R . \quad (3)$$

The image on the beam line is able to obtain by inverse Fourier transform of $\Phi(f)$.

3. Results and Discussion

We analyzed the singular values of the transfer matrix that has been related to the center frequency of the ultrasound, and examined the effect of number of the receivers and the singular values using image reconstruction. Transmit frequency is 3.5MHz, sampling frequency is 28.0MHz, number of transmitter is 64, and receivers is two type of 256, 1024. the field of view angle is the azimuthal angle $\pm30°$, the elevation angle is $\pm30°$.

Namely, number of subdivision of a field of view is 1024. Distance to the subject is 150mm.

Fig.1 and Fig.2 show the distribution of singular values. Number of the receivers is 256 and 1024.

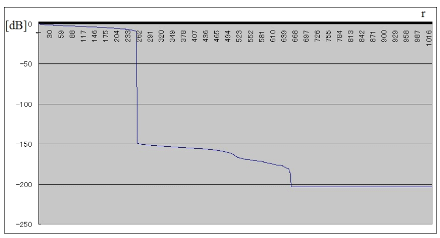

Fig.1 The distribution of singular values in 256 receivers.

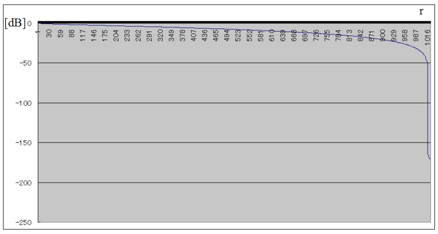

Fig.2 The distribution of singular values in 1024 receivers.

When number of singular values exceed number of receivers, the value is significantly reduced because the dimensions of matrix H is $N \times L$.

However, when 1024 receivers, the singular value decreases substantially before the number of divisions of imaging space reach 1024.

From these, when number of receivers is more than 1024, we found that not improve the image quality in the conditions of simulation that has been assumed no noise.

Fig.3, Fig.4, and Fig.5, Fig.6 show the PSF(Point Spread Function) when 256 receivers. Number of the singular values and regularization parameter are changed as shown in the figure.

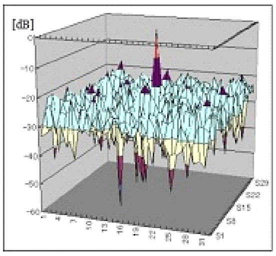

Fig.3 PSF of β=0,256 receivers and 256 singular values.

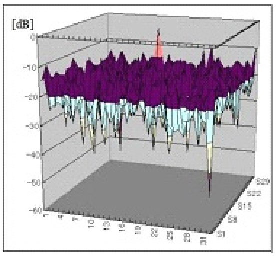

Fig.4 PSF of β=0,256 receivers and 1024 singular values.

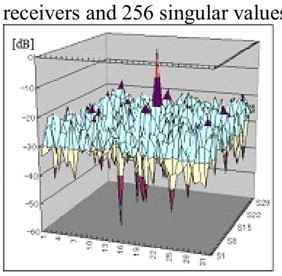

Fig.5 PSF of $\beta = 1.0 \times 10^{-11}$,256 receivers and 256 singular values.

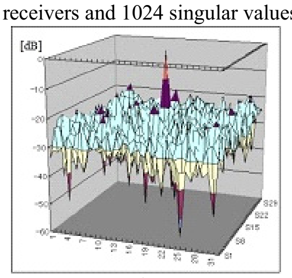

Fig.6 PSF of $\beta = 1.0 \times 10^{-11}$,256 receivers and 1024 singular values.

When β =0 and the above conditions, the second peak value in 256 and 1024 singular values are –16.5[dB] and –0.1[dB], respectively.

However, when $\beta = 1.0 \times 10^{-11}$, those are –16.5[dB] and –15.6[dB], respectively.

If number of the singular values exceed number of receivers, clearly, the second peak value amplify from the result of Fig.1. However, we found that be possible to suppress the noise by applying the regularization parameter.

Fig.7, Fig.8, Fig.9, and Fig.10 show the PSF when 1024 receivers. Number of the singular values and regularization parameter are changed as shown in the figure.

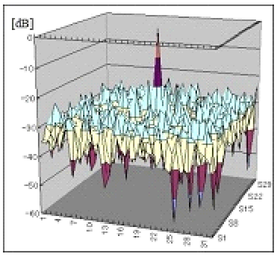

Fig.7 PSF of β=0,1024 receivers and 256 singular values.

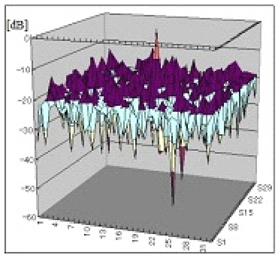

Fig.8 PSF of β=0,1024 receivers and 1024 singular values.

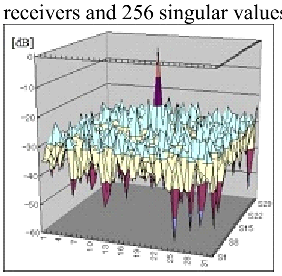

Fig.9 PSF of $\beta = 1.0 \times 10^{-11}$,1024 receivers and 256 singular values.

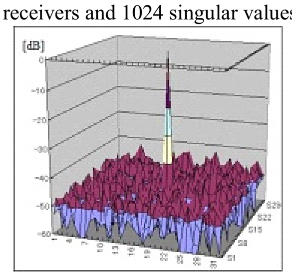

Fig.10 PSF of $\beta = 1.0 \times 10^{-11}$,1024 receivers and 1024 singular values.

When β =0 and the above conditions, the second peak value in 256 and 1024 singular values are –19.0[dB] and –10.1[dB], respectively.

However, when $\beta = 1.0 \times 10^{-11}$, those are –19.0[dB] and –40.6[dB], respectively.

As in the case of 256 receivers, we found that be possible to suppress the noise by applying the regularization parameter.

From these results, we could confirm the effectiveness of the regularization parameter. Also, we found that be possible to apply the method of the calculation that used a spatial pseudo-inverse matrix in the formation of a single frequency beam. In addition, it was suggested that be possible to determine the upper limit of number of elements of a sparse array from the distribution of singular values. In the future, we plan to clarify the characteristics of three-dimensional imaging calculation by inverse Fourier transform of a data in the frequency domain and to make the extension to the case of coded excitation.

References

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