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Role of Higher Modes of Guided Waves in Cylindrical Rods and Pipes

丸棒およびパイプを伝播するガイド波の高次モード

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1. Introduction

The mathematical formulation of guided waves propagating in cylindrical rods and pipes was derived by Pochhammer (1876) and Chree (1886) more than a century ago. It is called **P-C equation** in this paper.

Interests of early applications were mostly in the behavior of elongated structural materials. Hence beam and shell theories approximating the behavior of waves of long wavelength flourished.

Renewed interests aroused in the middle of the 20th century due to numerous new applicatins in electromechanical devices, such as ultasonic delay lines and mechanical filters, and in ultrasonic non-destructive testing of rods and pipes. The timely advent of electronic computers in the same period allowed exact numerical analyses of P-C equation, of which complexity had hindered manual computations.

P-C equation yields the relationship between frequency and wave number, which is called dispersion curves (branches). They are classified by the circumferenctial order, n, and the inharmonic overtone order, m, and are labeled as (n, m). In the axisymmetric mode, n = 0, curves are degenerated into the torsional modes, T(0, m), and the radial modes, R(0, m), which are mutually independent. Both modes allows the propagation down to zero frequency.

In the non-axisymmetric modes, only the flexural modes, n = 1, allows the propagation down to zero frequency. Other non-axisymmetric modes, n > 1, allows wave propagation at frequecy higher than the lowest cutoff frequency. This behavior is similar to cutoff phenomenon in a microwave waveguide. In microwave, it is known that an evanescent wave, of which amplitude exponentially decays with distance, exists even at frequecy lower than a cutoff frequency. It was found that P-C equation yielded similar non-propagating waves with purely imaginary wave number at first and then with complex wave number later.

Table 1 shows an overview of major contributions to solutions of P-C equation classified by parameter spaces covered.

Table 2 shows a similar overview for the special cases of plane strain and plane stress. The former yields cutoff frequencies (zero wave number) , which connect a real branch with a corresponding purely imaginary branch.

The latter is mathematically equivalent to the former, if Poisson's ratio is redefined, but lacks Longitudinal Shear modes contained in the former.

It can be seen in **Table 1** that the behavior of purely imaginary and complex branches for n > 1 modes of pipe have not been fully explored.

This paper presents detailed analyses of purely imaginary branches and in particular its contribution to trapped energy modes of vibration. Analyses of complex branches will follow in the next paper.

It is shown that trapped energy modes of vibration are closely related to the long lasting reverberation of bells and wine glasses.

2. Numerical Examples

Fig. 1 shows cutoff frequencies for σ =0.35 & n = 2 as functions of the ratio of outer and inner radius, Rab. It can be seen that the lowest three branches, (2,1) , LS(2,1) and (2,2), are well separated from other higher branches in thin pipes.

Fig. 2 shows a typical behavior of purely imaginary branches for Rab = 7/9 and 59/61. It can be seen that various trapped energy modes can be realized by slight changes in cross section of a pipe. Since these trapped energy modes can hardly propagate, a long lasting reverberation occurs when the pipe is struck. You may witness the phenomena when a bell or a wineglass rings.

References

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Notations

Ω = ω * b / Vs , Rab = a / b
ω : angular frequency, σ : Poisson's ratio
ξ : wave number along the axis
a: inner radius, b: outer radius
n: circumferential order m: inharmonic order
Vs: shear velocity

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	rod		
n	0	1	1 <
real wavenumber	Holland(1943)	Pao(1962)	
	Onoe(1955)		
	Onoe_M_M(1962)		
	Zemanek(1972)	Zemanek(1972)	Zemanek(1972)
purely imaginary wavenumber	Onoe(1955)	Pao(1962)	
	Onoe_M_M(1962)		
	Zemanek(1972)	Zemanek(1972)	Zemanek(1972)
complex wave number	Onoe_M_M(1962)	Pao(1962)	
	Zemanek(1972)	Zemanek(1972)	Zemanek(1972)
edge mode	Onoe(1961)		Libov(2012) n = 2
circumferential SH waves			
	pipe		
n	0	1	1 <
real wavenumber	GazisI&II(1959)	GazisI&II(1959)	GazisI&II(1959)
	Kanbe_T_A_T(1991)		
purely imaginary wavenumber	Kanbe_T_A_T(1991)	**Yet Unexplored**	
complex wave number			

Table 1　Overview of parameter spaces of solutions of PC equation.

plane strain			plane stress		
n = 0	**1**	**1 <**	**n = 0**	**1**	**1 <**
Gazis(1958)	Gazis(1958)	Gazis(1958)	Onoe(1956)	Onoe(1956)	Onoe(1956)
					Onoe(1958) n = 2
			Holland(1966)	Holland(1966)	Holland(1966)
			Nakamura_S(1972)	Nakamura_S(1972)	Nakamura_S(1972)

Table 2　Overview of parameter spaces of solutions of PC equation for 2D plane strain and plane stress

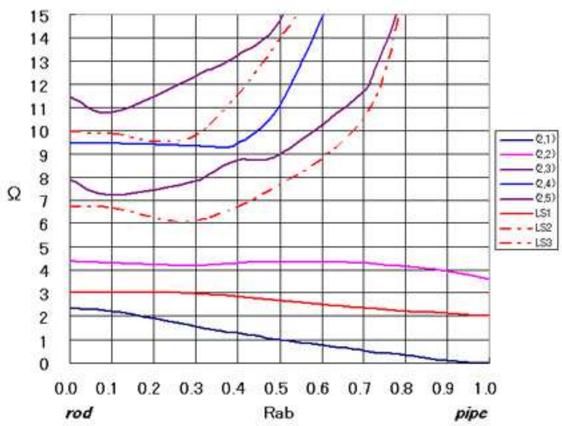

Fig. 1 Cutoff frequencies for σ =0.35, n =2

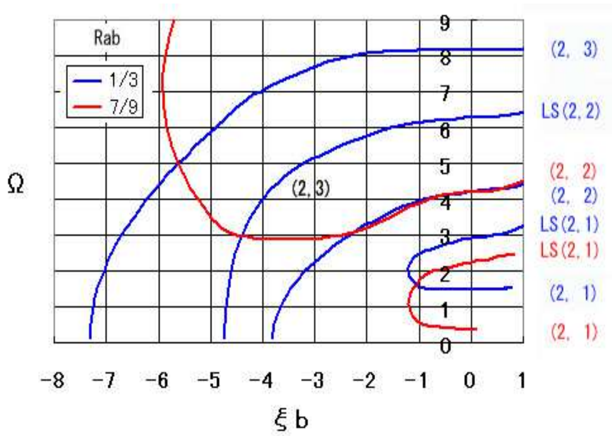

Fig. 2 Dispersion curves for σ =0.35, n =2