

Study on the birefringence of ultrasonic transverse wave in [110] silicon

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1. Introduction

Wave velocity in anisotropic solids depends on propagation direction as well as on the direction of particle displacement. Wave velocities with corresponding direction of particle displacement can be determined by Christoffel's equation for given direction of propagation [1]. In this research, transverse wave decomposition has been investigated when the transverse wave with various polarization propagates in [110] direction of silicon.

Christoffel's equation to determine wave-velocities in an anisotropic elastic solid is given as following:

$$\begin{vmatrix} (\lambda_{11} - \rho c^2) & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & (\lambda_{22} - \rho c^2) & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & (\lambda_{33} - \rho c^2) \end{vmatrix} = 0$$

where $\lambda_{im} = c_{iklm}n_k n_l$. c_{iklm} , n_k , c and ρ is elastic constants of solid, direction cosine, wave velocity, and density of the solid, respectively. Since silicon is cubic crystal and elastic constants are well-known, wave velocities and corresponding direction of particle displacement can be easily obtained by solving Christoffel's equation. **Table 1** shows ultrasonic waves propagating in [110] direction of silicon.

[110] Silicon	c_1	c_2	c_3
Wave velocity(m/s)	9133.8	5842.9	4672.6
Wave Type	Longitudinal	Transverse	Transverse
Direction of particle displacement	Parallel	[001]	[1 $\bar{1}$ 0]

Table 1 Wave velocity and direction of particle displacement for ultrasonic waves in [110] direction of silicon. [3]

In case of [110] direction, one longitudinal wave and two transverse waves can exist. It is noticeable that two possible transverse wave modes can propagate in [110] direction, and the velocities depend on the direction of particle displacement

(called as '*polarization*' from now). These two transverse wave components act as principal axes of polarization. Transverse wave with arbitrary polarization is decomposed into two modes of transverse wave with wave velocities of c_1 and c_2 and they will simultaneously propagate with different velocities. Angle between two component axes is almost perpendicular. ($\approx 89^\circ$)

3. Experimental Setup

The specimen used in this work is [110] silicon disk with thickness of 8mm and diameter of 5cm. Transverse wave is generated into the disk by the transducer whose frequency and diameter are about 2.5 MHz and 18mm, respectively. The specimen is rotated on the optical rotating stage so that polarization angle can be accurately changed. Echo waveforms were measured for the polarization of incident wave from [001] axis to [1 $\bar{1}$ 0] axis for 10° step.

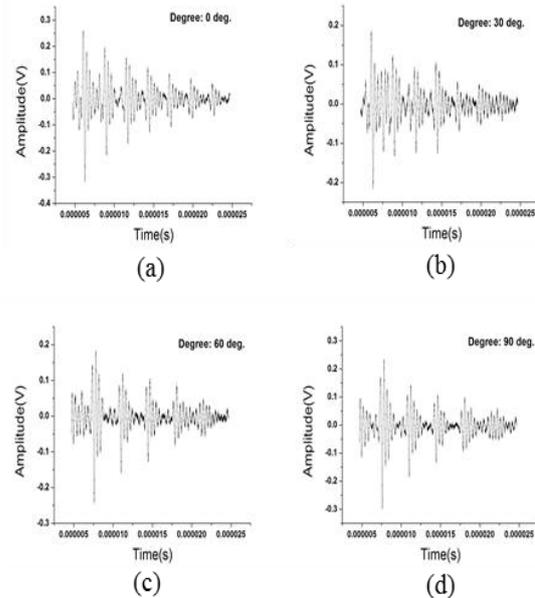


Figure 1 Waveforms for various polarization angles; (a) 0 degree ([001] mode), (b) 30 degrees, (c) 60 degrees and (d) 90 degrees ([1 $\bar{1}$ 0] mode)

Acquired data are corrected under the consideration of transmission and reflection of the wave at the boundary.

4. Results

4.1. Effect of polarization angle on amplitude of two transverse wave modes

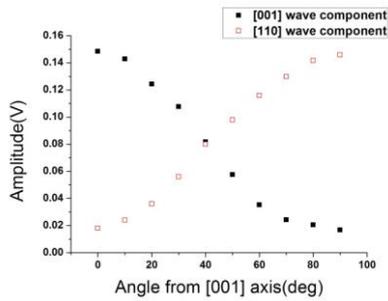


Figure 2 Relation between amplitude of two wave modes and angle from [001] axis.

Wave amplitude depends on the condition of contact between a transducer and a disk. In this experiment, pressure is applied on the transducer so that the contact is strong enough to make the amplitude maximized.

Since transverse wave penetrating through [110] direction can only have two modes, wave decomposition is observed for the wave with arbitrarily polarization. (Figure 2) As initial polarization of transverse wave becomes near [110] mode axis, [001] mode diminishes whereas [110] mode strengthened.

In contrast to general optical polarization, two decomposed modes have the same amplitude at about 40°. Moreover, even though two principal axes are almost perpendicular, summation of two squared amplitude values is not constant.

4.2. Deviation of transverse wave splitting from normal vector decomposition

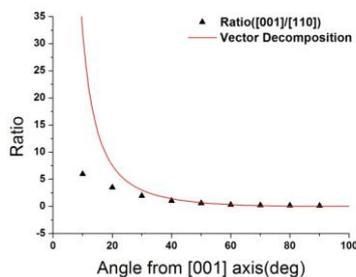


Figure 3 Comparison of amplitude ratio to vector decomposition

Figure 3 shows deviation of amplitude ratio ($[001]/[1\bar{1}0]$) from the ratio expected by vector decomposition. Deviation from vector decomposition ratio is remarkable when the polarization of transverse wave is near [001] mode.

It is thought that velocity difference between two principal axes mainly contributes to this deviation. Especially, deviation looks stronger near [001] axis, and this might be because the velocity of [001] wave mode is faster than that of the other. That is, transverse wave mode with faster wave velocity tends to have smaller amplitude than vector estimation.

5. Conclusion

In the present work, birefringence of transverse wave in [110] silicon was investigated. Transverse wave with arbitrary polarization decomposed into two wave modes estimated by Christoffel's equation, but it showed deviation from vector decomposition of amplitude. Possible cause for this phenomenon is velocity difference between two transverse wave modes.

Acknowledgement

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References

- [1] H. F. Pollard: *Sound Waves in Solids* (Pion, London, 1977) p. 29-30.
- [2] F. Tseng and K. Chang: *J. Micromech. Microeng.* **13**(2003) 47-52
- [3] M. Kang, B. Jun and Y. H. Kim: *6th NDT in Progress* (2011) p. 2.
- [4] L. E. Kinsler, A. R. Frey, A. B. Coppens and J. V. Sanders: *Fundamentals of Acoustics* (Wiley, NJ, 2000) p. 149-152.