2P2-5

Proceedings of Symposium on Ultrasonic Electronics, Vol. 33 (2012) pp. 177-178
13-15 November, 2012

Analysis of guided wave which propagates pipe or pipe with fluid with attenuation

パイプを伝搬するガイド波の減衰を考慮した解析

Harumichi Sato†, and Hisato Ogiso (AIST)
佐藤治道†，小木曽久人 (産総研)

1. Introduction

Cylindrical pipes are widely used in industries such as nuclear power plants and micro total analysis systems (μTAS). Nondestructive evaluation (NDE) of such pipes is therefore crucial. NDE as well as ultrasonic flowmeters can be used to characterize pipes filled with fluid. Guide wave of a hollow pipe was investigated theoretically by Gazis[1], and we previously expanded on the theory proposed by Gazis for a fluid-filled pipe[2,3]. Those studies were for the condition that attenuations of the pipe and the fluid are negligibly small. However, the attenuations can not be neglected in some condition such as a inspection of a spallation neutron source mercury target, and a nondestructive inspection of an erosion of the mercury container walls[4] is required. Threfore, we analyzed the guided wave which propagates pipe with attenuation.

2. Theoretical analysis

Fig. 1 shows the theoretical model of a cylindrical pipe and its coordinate system (cylindrical coordinates). The author's theoretical basis is an expansion of that of hollow pipe by Gazis[1]. The displacement $\mathbf{u}^{\text{solid}}$ of the pipe ($a \le r \le b$) and the displacement $\mathbf{u}^{\text{fluid}}$ of a fluid ($0 \le r \le a$) are represented by a vector ($\mathbf{H}$) and scalar potential (ϕ_s, ϕ_f) as follows.

$$\mathbf{u}^{\text{solid}} = \nabla\phi_s + \nabla\times\mathbf{H} \qquad (1)$$

$$\mathbf{u}^{\text{fluid}} = \nabla\phi_f$$

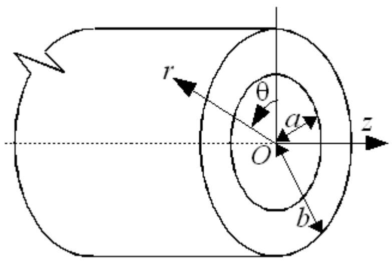

Fig. 1 Theoretical model

Wave equations of the potentials are as follows.

$$v_l^2\nabla^2\phi_s = \frac{\partial^2\phi_s}{\partial t^2}$$

$$v_t^2\nabla^2\mathbf{H} = \frac{\partial^2\mathbf{H}}{\partial t^2} \qquad (2)$$

$$v_f^2\nabla^2\phi_f = \frac{\partial^2\phi_f}{\partial t^2}$$

Here, t idicates time, v_l, v_t and v_f represent sound velocities of longitudinal wave of pipe, taransversal wave of pipe and logitudinal wave of fluid, respectively. The potentials are as follows.

$$\phi_s = f_s(r)\cos n\theta \exp i(k^* z - \omega t)$$
$$H_r = g_r(r)\sin n\theta \exp i(k^* z - \omega t + \pi/2)$$
$$H_\theta = g_\theta(r)\sin n\theta \exp i(k^* z - \omega t + \pi/2)$$
$$H_z = g_3(r)\sin n\theta \exp i(k^* z - \omega t)$$
$$\phi_f = f_f(r)\cos n\theta \exp i(k_f{}^* z - \omega t) \qquad (3)$$
$$k^* = k(1+i\eta),\quad k_f{}^* = k_f(1+i\eta_f)$$
$$k_f = \frac{\omega}{V-v},\quad V = \frac{\omega}{k}$$

k, ω, n, i, η, η_f and v represente the wave number of the guided wave propagating in a pipe, the angular frequency, the circumferential mode parameter, the imaginary unit and an attenuation constant of the pipe, an attenuation constant of the fluid and a flow velocity of the fluid, respectively. By eq.(2) and (3), below equations are obtained.

$$f_s = A_s Z_n(\alpha_1 r) + B_s W_n(\alpha_1 r)$$
$$g_3 = A_3 Z_n(\beta_1 r) + B_3 W_n(\beta_1 r)$$
$$2g_1 = (g_r - g_\theta)$$
$$= 2A_1 Z_{n+1}(\beta_1 r) + 2B_1 W_{n+1}(\beta_1 r) \qquad (4)$$
$$2g_2 = (g_r + g_\theta)$$
$$= 2A_2 Z_{n-1}(\beta_1 r) + 2B_2 W_{n-1}(\beta_1 r)$$
$$f_f = A_f Z_n(\chi_1 r)$$

$$\alpha^2 = \omega^2 / v_l^2 - k^{*2}$$
$$\beta^2 = \omega^2 / v_t^2 - k^{*2} \qquad (5)$$
$$\chi^2 = \omega^2 / v_f^2 - k_f{}^{*2}$$

J_n, Y_n, I_n, K_n are the Bessel function of the

---

h.sato@aist.go.jp

first kind, the Bessel function of the second kind, the modified Bessel function of the first kind and the modified Bessel function of the second kind, respectively. Z_n, W_n, α_1, β_1 and χ_1 are show in Table 1-3. Each argument of the Bessel functions become a complex number when η or η_f is not zero. By the property of the gauge invariance, any one of the three potentials g_i ($i = 1$, 2, or 3) can be set to zero. Setting $g_2 = 0$ yields

$$g_r = -g_\theta = g_1 \tag{6}$$

By eq.(1), (3) and (6), the displacements are as follows.

$$\begin{aligned} u_r^{\mathrm{solid}} &= [f_s' + (n/r)g_3 - k^* g_1] \\ &\quad \times \cos n\theta \exp i(k^* z - \omega t) \\ u_\theta^{\mathrm{solid}} &= [-(n/r)f_s - k^* g_1 - g_3'] \\ &\quad \times \sin n\theta \exp i(k^* z - \omega t) \\ u_z^{\mathrm{solid}} &= [k^* f_s - g_1' - (n+1)(g_1/r)] \\ &\quad \times \cos n\theta \\ &\quad \times \exp i(k^* z - \omega t + \pi/2) \\ u_r^{\mathrm{fluid}} &= f_f' \cos n\theta \exp i(k_f^* z - \omega t) \end{aligned} \tag{7}$$

The boundary conditions are as follows.

$$\begin{aligned} &u_r^{\mathrm{solid}} = u_r^{\mathrm{fluid}}, \quad \sigma_{rr}^{\mathrm{solid}} = \sigma_{rr}^{\mathrm{fluid}}, \\ &\sigma_{r\theta}^{\mathrm{solid}} = \sigma_{rz}^{\mathrm{solid}} = 0 \quad \text{at} \quad r = a \\ &\sigma_{rr}^{\mathrm{solid}} = \sigma_{r\theta}^{\mathrm{solid}} = \sigma_{rz}^{\mathrm{solid}} = 0 \quad \text{at} \quad r = b \end{aligned} \tag{8}$$

$\boldsymbol{\sigma}^{\mathrm{solid}}$ and $\boldsymbol{\sigma}^{\mathrm{fluid}}$ are the stress tensors of the pipe and fluid, respectively. They are obtained by displacements and solid's and fluid's densities (ρ_s and ρ_f). By eq. (7) and (8), a homogeneous systems of linear equations is obtained.

$$\begin{aligned} &[c_{ij}]\mathbf{x} = 0 \\ &{}^t\mathbf{x} = (A_s, A_1, A_3, B_s, B_1, B_3, A_f) \end{aligned} \tag{9}$$

$[c_{ij}]$ is a 7×7 matrix, and c_{ij} are similar to our previous result except for k and k_f. All k and k_f of c_{ij} in refs. 2 and 3 are replaced by k^* and k_f^*, respectively. For example, some c_{ij} s are shown below.

$$\begin{aligned} c_{12} &= k^* a Z_{n+1}(\beta_1 a) \\ c_{17} &= [-nZ_n(\chi_1 a) + \chi_1 \lambda_\chi a Z_{n+1}(\chi_1 a)] \\ &\quad \times \exp i(k_f^* z - \omega t) / \exp i(k^* z - \omega t) \\ c_{21} &= [2n(n-1) - (\beta^2 - k^{*2})a^2] Z_n(\alpha_1 a) \\ &\quad + 2\lambda_1 \alpha_1 a Z_{n+1}(\alpha_1 a) \\ c_{27} &= [\rho_f \omega^2 a^2 Z_n(\chi_1 a) / (\rho v_t^2)] \\ &\quad \times \exp i(k_f^* z - \omega t) / \exp i(k^* z - \omega t) \end{aligned} \tag{10}$$

A nontrivial solution is obtaiend when the determinant of $[c_{ij}]$ is zero.

$$\det[c_{ij}] = 0 \tag{11}$$

Because eq (11) containes the frequency ($f = \omega / 2\pi$) and the phase velocity (V), the dispersion curves are obtained.

Table 1 Parameters for α

	α_1	λ_α	$Z_n(\alpha_1 r)$	$W_n(\alpha_1 r)$
$\mathrm{Re}(\alpha^2) > 0$	α	1	$J_n(\alpha_1 r)$	$Y_n(\alpha_1 r)$
$\mathrm{Re}(\alpha^2) < 0$	α / i	-1	$I_n(\alpha_1 r)$	$K_n(\alpha_1 r)$

Table 2 Parameters for β

	β_1	λ_β	$Z_n(\beta_1 r)$	$W_n(\beta_1 r)$
$\mathrm{Re}(\beta^2) > 0$	β	1	$J_n(\beta_1 r)$	$Y_n(\beta_1 r)$
$\mathrm{Re}(\beta^2) < 0$	β / i	-1	$I_n(\beta_1 r)$	$K_n(\beta_1 r)$

Table 3 Parameters for χ

	χ_1	λ_χ	$Z_n(\chi_1 r)$
$\mathrm{Re}(\chi^2) > 0$	χ	1	$J_n(\chi_1 r)$
$\mathrm{Re}(\chi^2) < 0$	χ / i	-1	$I_n(\chi_1 r)$

3. Discussions and Conclusions

We obtained anyalitical result of the guided wave which propagates pipe or pipe with fluid with attenuation. As a sample, the determinant of $[c_{ij}]$ is plotted in **Fig. 2**. We can see two V s in **Fig. 2**.

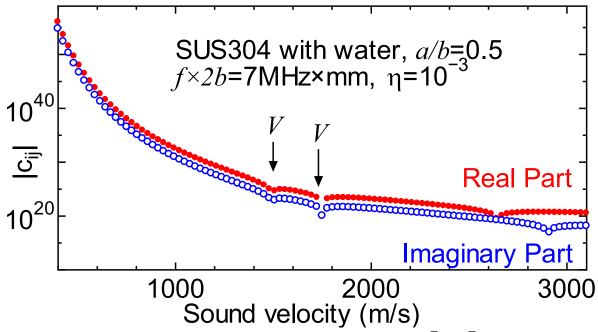

Fig. 2 Determinant of $[c_{ij}]$

Acknowledgment

This work was supported by MEXT/JSPS KAKENHI Grant Number 23561021.

References

1. D. C. Gazis: J. Acoust. Soc. Am. **31** (1959) 568.
2. H. Sato, M. Lebedev, and J. Akedo: Jpn. J. Appl. Phys. **45** (2006) 4573.
3. H. Sato, M. Lebedev, and J. Akedo: Jpn. J. Appl. Phys. **46** (2007) 4521[Errata; **47** (2008) 403].
4. J. R. Haines *et al.*: J. Nucl. Mater. **343** (2005) 58.