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Interfacial Elastic Waves Propagating along the Interface in a Dual Two-dimensional Phononic-Crystal System

異なる二つの２次元フォノニック結晶界面を伝播する界面波の解析

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1. Introduction

Surface acoustic waves (SAWs) are confined in the surface, and then it is possible to excite and detect the SAWs on the surface. An interdigital transducer (IDT) attached on the surface can convert the SAWs to microwaves and vice versa, which is used in signal processing. Thus the advantages of the SAWs are exploited in the SAW device, however, the localization in the surface leads simultaneously to the shortcomings of the SAWs; the propagation is very sensitive to surface roughness and to disturbance from the environment onto the surface. Then the surface needs to be protected from damage or disturbance. If the acoustic waves are localized beneath the surface, such devices will have various uses. An acoustic wave free from the shortcomings is an interfacial acoustic wave (IAW). The waves are localized at the interface between two dissimilar materials. A material with a thin layer on the surface may support the IAWs, and then it will be possible to excite and detect the IAWs with the IDT, leading to an IAW device. However, such a structure does not always support the IAWs since the existence of the IAWs depends on the combination of the two materials [1].

Although the intrinsic material parameters cannot be changed, we can control the material parameters of composite materials. In the present work, we investigate existence and propagation of IAW in the interface between two composite materials. We utilize two-dimensional (2D) phononic crystals (PCs), where 2D arrays of cylinders are embedded in a matrix. The material parameters of PCs can be adjusted by choosing proper materials. Since the PCs have periodic structures, the system is expected to have the band structure for IAWs. That means the propagation of IAWs can be controlled with a high degree of freedom.

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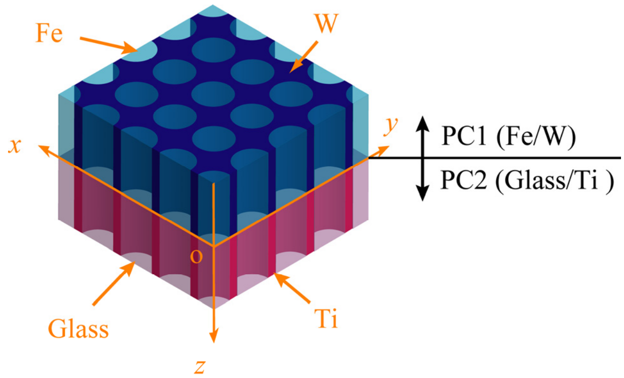

FIG1: Dual two-dimensional phononic-crystal system. The normal vector of the interface between the 2DPCs is parallel to the axes of cylinders. The PC1 (PC2) is composed of iron (glass) cylinders and tungsten (titanium) matrix with the filling fraction f_1 (f_2), which is denoted by ‘Fe/W (Glass/Ti) PC’.

2. Model and Methodology

We consider the system composed of two semi-infinite 2D PCs (denoted by PC1 and PC2) as shown in Fig. 1, which is referred to as a *dual 2D PC system*. The normal vector of the interface between the 2DPCs is parallel to the axes of cylinders. The PC1 (PC2) is composed of iron (glass) cylinders embedded in tungsten (titanium) matrix with the filling fraction f_1 (f_2), which is referred to as ‘Fe/W (Glass/Ti) PC’. Using a plane-wave-expansion (PWE) method [2-4], we investigate the existence and propagation characteristics of IAWs at the interface between the 2D Fe/W and the 2D Glass/Ti PC.

To begin with, we obtain the displacement vector $\mathbf{u}(\mathbf{r},t)$ in each PC. The equation of motion for the displacement vector $\mathbf{u}(\mathbf{r},t)$ yields

$$\rho(\mathbf{r})\ddot{u}_i(\mathbf{r},t)=\partial_j c_{ijkl}(\mathbf{r})\partial_k u_l(\mathbf{r},t) \quad (i=x,y,z) \quad (1)$$

where $\rho(\mathbf{r})$ and $c_{ijkl}(\mathbf{r})$ are the mass density and elastic stiffness tensor, respectively. They vary periodically in the x-y plane, and then are expanded in a Fourier series as

$$\rho(\mathbf{x}) = \sum_{\mathbf{G}} \rho_{\mathbf{G}} e^{i\mathbf{G}\cdot\mathbf{x}} , \qquad (2)$$

$$c_{ijkl}(\mathbf{x}) = \sum_{\mathbf{G}} C^{ijkl}_{\mathbf{G}} e^{i\mathbf{G}\cdot\mathbf{x}} , \qquad (3)$$

where $\mathbf{G} = (G_x, G_y)$ is a 2D reciprocal lattice vector and $\mathbf{x} = (x, y)$ is a 2D vector in the *x-y* plane. Since $\mathbf{u}(\mathbf{r}, t)$ is subject to the Bloch's theorem, the displacement field $\mathbf{u}(\mathbf{r}, t)$ is given by

$$\mathbf{u}(\mathbf{r},t) = e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \sum_{\mathbf{G}} \mathbf{U}_{\mathbf{G}} e^{i\mathbf{G}\cdot\mathbf{x}} , \qquad (4)$$

where $\mathbf{k} = (\mathbf{k}_{\parallel}, k_z)$ is a 3D wave vector, and ω is an angular frequency. Substituting Eqs.(2), (3) and (4) into Eq. (1), we obtain the secular equation. Once ω and $\mathbf{k}_{\parallel} = (k_x, k_y)$ being given, the *z*-component of a wave vector k_z is derived as an eigenvalue from the secular equation. We simultaneously obtain the eigen displacement vector at $(\mathbf{k}, \omega)$ in each PC.

When the two PCs are connected, the eigen modes in each PC do not satisfy boundary condition at the interface given below. We express the displacement vector in terms of a linear combination of the eigen vectors in each PC;

$$\mathbf{u}(\mathbf{r},t) = e^{i(k_x x + k_y y - \omega t)} \sum_{\lambda} A_{\lambda} e^{ik_z^{(\lambda)} z} \sum_{\mathbf{G}} \mathbf{U}^{(\lambda)}_{\mathbf{G}} e^{i\mathbf{G}\cdot\mathbf{x}} . \qquad (5)$$

and determine the coefficients A_{λ} so that Eq.(5) satisfies the boundary condition at the interface.

The boundary conditions (BCs) that the IAWs satisfy is matching of the displacement vectors and stresses at the interface. In addition, the displacement vector decays exponentially with leaving from the interface so that the mode is confined at the interface. Mathematically, the exponential decay is brought by a pure imaginary eigenvalue ($k_z = -i\kappa\,(+i\kappa)$). A set of ω and $\mathbf{k}_{\parallel}$ that satisfy all the conditions gives the dispersion relation $\omega = \omega(\mathbf{k}_{\parallel})$ of IAWs.

3. Numerical Results

Figure 2(a) plots the dispersion relation of IAWs in the dual 2D PC system composed of Fe/W PC with f_1=0.2 and Glass/Ti PC with f_2=0.3. We found that the combination of these materials supports IAWs at the interface between the two PCs. In addition, the group velocity of the IAW decreases near the boundary of the Brillouin zone due to the periodicity of PCs.

In order to elucidate the behavior of IAWs near the interface, Figure 2(b) shows the amplitude distribution of the IAW corresponding to the point denoted by the arrow in Fig. 2(a). The polarization of IAWs is in the sagittal plane (*x-z* plane) and there is a phase difference by $\pi/2$ between U_z and U_x, leading to an elliptic particle motion like the Rayleigh waves. Because there is a node at z/a=-0.8 in Fe/W PC, the elliptic motion changes the direction at the position. The amplitude of IAWs is also found to be distributed mainly in the side of the PC (Fe/W PC).

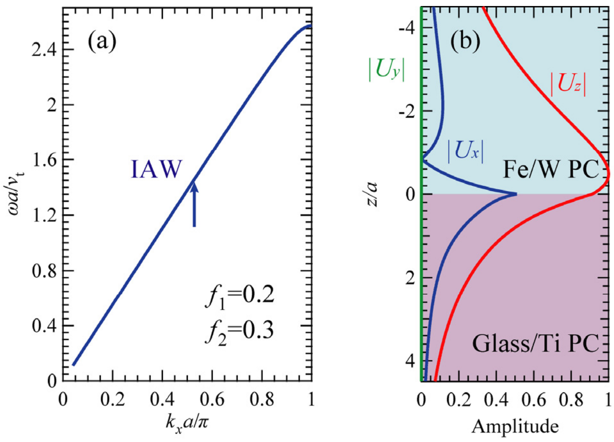

FIG2: (a) Dispersion relation of interfacial acoustic wave in a interface between a 2D Fe/W PC and a 2D Glass/Ti PC. The filling fraction f_1 of the Fe/W PC is 0.2, f_2 of the Glass/Ti PC is 0.3. (b) The amplitude components of the interfacial acoustic wave at the wavenumber denoted by the arrow in Figure 2(a).

4. Concluding Remarks

We confirmed numerically the IAW in the dual 2D PC systems for the specific combination of filling fractions of two PCs. We find the almost linear dispersion relation as well as the elliptic particle motion similar to the Rayleigh waves. We expect that the findings of this work lead to the novel device applications of phononic crystals.

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