

Propagation of Sound Wave in Rarefied Polyatomic Gases Based on Extended Thermodynamics I. Longitudinal Wave

拡張された熱力学に基づく多原子分子希薄気体中を伝播する音波の解析 I. 縦波

Takashi Arima^{1†}, Shigeru Taniguchi¹, Tommaso Ruggeri² and Masaru Sugiyama¹
(¹Nagoya Inst. of Tech.; ²Univ. of Bologna)

有馬隆司^{1†}, 谷口茂¹, ルージェリ・トマソ², 杉山勝¹(¹名工大, ²ボローニャ大学)

1. Introduction

Extended thermodynamics (ET) [1] is a phenomenological theory which can describe a nonequilibrium phenomena with evident spatio-temporal changes of physical quantities beyond the assumption of local equilibrium. Recently ET theory for rarefied polyatomic gases and also dense gases is established [2]. The purpose of the present paper is to make clear the validity and feature of this theory through studying the propagation of sound wave. The dispersion relation is derived and compared with experimental data and those by Navier-Stokes Fourier theory (NSF) [3] which is based on the assumption of local equilibrium.

2. Basic equations

We study rarefied polyatomic gases with the equations of state for the pressure $p = a\rho T$ and the specific internal energy $\varepsilon = \varepsilon(T)$, where ρ and T are the mass density and the temperature, respectively, and $a = k_B / m$ with k_B and m being the Boltzmann constant and the mass of a molecule.

We assume that a nonequilibrium state can be characterized by 14 independent fields $\mathbf{u} = (\rho, v_i, T, S_{<ij>}, \Pi (=S_{ii}/3), q_i)$ ($i, j = 1, 2, 3$), where $v_i, S_{<ij>}, \Pi$ and q_i are the velocity, symmetric trace part of viscous stress, dynamic pressure and heat flux, respectively. Here the summation convention is adopted. Let us consider the linearized system in the neighborhood of a constant equilibrium state $\mathbf{u}_0 = (\rho_0, 0, T_0, 0, 0, 0)$. The system for perturbed fields \mathbf{u} are expressed as follows:

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial v_k}{\partial x_k} &= 0, \\ \rho_0 \frac{\partial v_i}{\partial t} + aT_0 \frac{\partial \rho}{\partial x_i} + a\rho_0 \frac{\partial T}{\partial x_i} - \frac{\partial S_{<ij>}}{\partial x_j} + \frac{\partial \Pi}{\partial x_i} &= 0, \\ a\rho_0 c_v^* \frac{\partial T}{\partial t} + a\rho_0 T_0 \frac{\partial v_k}{\partial x_k} + \frac{\partial q_k}{\partial x_k} &= 0, \\ \frac{\partial S_{<ij>}}{\partial t} - 2a\rho_0 T_0 \frac{\partial v_{<i}}{\partial x_{j>}} - \frac{2}{1+c_v^*} \frac{\partial q_{<i}}{\partial x_{j>}} &= -\frac{1}{\tau_S} S_{<ij>}, \end{aligned}$$

$$\begin{aligned} \frac{\partial \Pi}{\partial t} + \left(\frac{2}{3} - \frac{1}{c_v^*} \right) a\rho_0 T_0 \frac{\partial v_k}{\partial x_k} + \frac{2c_v^* - 3}{3c_v^*(1+c_v^*)} \frac{\partial q_k}{\partial x_k} &= -\frac{1}{\tau_\Pi} \Pi, \\ \frac{\partial q_i}{\partial t} + (1+c_v^*) a^2 \rho_0 T_0 \frac{\partial T}{\partial x_i} - aT_0 \frac{\partial S_{<ik>}}{\partial x_k} + aT_0 \frac{\partial \Pi}{\partial x_i} &= -\frac{1}{\tau_q} q_i, \end{aligned}$$

where t, x_i and c_v^* are, respectively, time, position and the dimensionless specific heat defined by $(d\varepsilon/dT)_{T=T_0}/a$. The value of c_v^* can be estimated by experimental data or statistical mechanical study [4]. The relaxation times τ_S, τ_Π and τ_q are related to the shear viscosity μ , bulk viscosity ν and heat conductivity κ as follows:

$$\mu = a\rho_0 T_0 \tau_S, \nu = \left(\frac{2}{3} - \frac{1}{c_v^*} \right) a\rho_0 T_0 \tau_\Pi, \kappa = (1+c_v^*) a^2 \rho_0 T_0 \tau_q.$$

3. Dispersion relation for sound wave

Let us consider a plane harmonic wave propagating in x_1 -direction with frequency ω and complex wave number k such that $\mathbf{u} = \mathbf{w} \exp(i(\omega t - kx_1))$ where \mathbf{w} is a constant amplitude vector. We assume a longitudinal wave:

$$v_i \equiv \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix}, S_{<ij>} \equiv \begin{pmatrix} S & 0 & 0 \\ 0 & -S/2 & 0 \\ 0 & 0 & -S/2 \end{pmatrix}, q_i \equiv \begin{pmatrix} q \\ 0 \\ 0 \end{pmatrix}.$$

Then the dispersion relation is obtained as follows:

$$\begin{aligned} \frac{c_v^* (c_0 z)^4}{3\Omega^2 (1+c_v^*)^2} \left(9\Omega^2 c_v^* - i\Omega \left(\frac{3+7c_v^*}{\tau_{ps}} + 5c_v^* \right) - 3\frac{1+c_v^*}{\tau_{ps}} \right) \\ + \frac{c_v^* (c_0 z)^2}{3\Omega^3 (1+c_v^*)^2} \left[-3\Omega^3 c_v^* (7+4c_v^*) \right. \\ \left. + i\Omega^2 \left(2 \left(\frac{3+10c_v^*+5c_v^{*2}}{\tau_{ps}} \right) + 9c_v^* \frac{1+c_v^*}{\tau_{qs}} + c_v^* (13+8c_v^*) \right) \right. \\ \left. + \Omega (1+c_v^*) \left(\frac{3+7c_v^*}{\tau_{ps}\tau_{qs}} + \frac{5c_v^*}{\tau_{qs}} + 6\frac{1+c_v^*}{\tau_{ps}} \right) - 3i \frac{(1+c_v^*)^2}{\tau_{ps}\tau_{qs}} \right] \\ + \frac{(\Omega - i)(\tau_{ps}\Omega - i)(\tau_{qs}\Omega - i)}{\Omega^3 \tau_{qs} \tau_{ps}} = 0, \end{aligned}$$

where $z=k/\omega$, $\Omega=\tau_S\omega$, $\tau_{qs}=\tau_q/\tau_S$, $\tau_{ps}=\tau_{\Pi}/\tau_S$ and the sound velocity in equilibrium defined by $c_0=\sqrt{\gamma T_0(1+1/c_v^*)}$. With the help of the experimental data on μ, ν and κ , we can, in principle, estimate the temperature dependence of τ_{qs} and τ_{ps} . However, at present, as we have the reliable data only on μ and κ [5], we adopt $\varphi=\nu/\mu$ as an adjustable parameter.

4. Comparison with experimental data

From the dispersion relation we study the phase velocity v_{ph} and attenuation factor α which are calculated as $v_{ph}(\omega)=\omega/\text{Re}(k)=1/\text{Re}(z)$ and $\alpha(\omega)=-\text{Im}(k)=-\omega\text{Im}(z)$ for hydrogen, deuterium and hydrogen deuteride gases. The predictions by ET are compared with experimental data and those by NSF.

For normal hydrogen (n-H₂), the dependence of the dimensionless phase velocity v_{ph}/c_0 and dimensionless attenuation factor $c_0\tau_S\alpha$ on the reduced frequency Ω are shown in Fig. 1. We see the experimental data on the phase velocity at $T_0=273.5, 296.8$ [K] by Rhodes [6] and on the attenuation factor at $T_0=293$ [K] by Sluijter et al. [5], and the theoretical predictions by ET and NSF at $T_0=293$ [K]. We evaluate $c_v^*=2.45$, $c_0=1300$ [m/s], $\mu=8.82$ [$\mu\text{Pa}\cdot\text{s}$], $\kappa=182$ [mW/(m·K)], $\tau_{qs}=1.46$, and moreover we estimate the relaxation times as $\tau_S p_0=8.82$ [s· μPa] and $\tau_{\Pi} p_0=12.8$ [s· μPa] where p_0 is the pressure at the reference equilibrium state.

In the region with small Ω the predictions by ET and NSF coincide with each other and φ is selected as 37 to fit these results with the experimental data. Therefore $\nu=326$ [s· μPa], $\tau_{ps}=144$ and $\tau_{\Pi} p_0=1270$ [s· μPa]. When Ω becomes large the prediction by the ET theory is evidently superior to that by the NSF theory. The difference between the two theories emerges around $\Omega=10^{-3}$. Furthermore the ET theory shows good fitting with experimental data up to $\Omega=10^{-1}$.

For other temperature and for other gases the details are shown in the paper [7].

5. Summary and concluding remarks

We have seen clearly that the recent theory of ET is consistent with the experimental data even in the high frequency range where NSF is no more valid. We have also evaluated the bulk viscosity and the relaxation times.

Although the comparisons with the experimental data are entirely-focused on rarefied diatomic gases, the consistency strongly convinces us of the validity of the present theory not only for polyatomic gases, but also for dense gases.

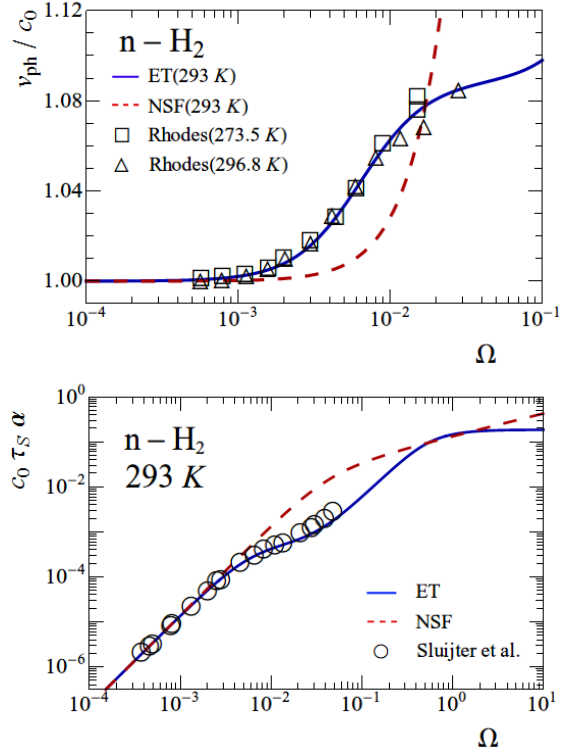


Fig. 1 Dependence of the dimensionless phase velocity (upper) and dimensionless attenuation factor (lower) on the reduced frequency Ω for n-H₂. The squares and triangles in upper figure are the experimental data of phase velocity at $T_0=273.5$ and 296.8 [K], respectively, by Rhodes [6] and the circles in the lower figure are those at $T_0=293$ [K] by Sluijter et al.[5]. The solid and dashed lines are, respectively, predictions by the ET and NSF theories at 293[K] and φ is selected as 37.

Acknowledgment

This work was partially supported by: Japan Society of Promotion of Science (JSPS) No. 24760055 (S.T.), No. 20560054 (M.S.); MIUR/PRIN Project “Nonlinear Propagation and Stability in Thermodynamical Processes of Continuous Media”(T.R.).

References

1. I. Müller and T. Ruggeri: *Rational Extended Thermodynamics* (Springer, New York, 1998) 2nd ed.
2. T. Arima, S. Taniguchi, T. Ruggeri and M. Sugiyama: *Cont. Mech. Thermodyn.* DOI: 10.1007/s00161-011-0213-x (2011) 1.
3. S. R. De Groot and P. Mazur: *Non-Equilibrium Thermodynamics* (North-Holland, Amsterdam, 1963).
4. L. D. Landau and E. M. Lifshitz: *Statistical Physics* (Oxford, Pergamon, 1980).
5. C. Sluijter, H. Knaap and J. Beenakker: *Physica.* **30** (1964) 745.
6. E. J. Rhodes JR: *Phys. Rev.* **70**[11] (1946) 932.
7. T. Arima, S. Taniguchi, T. Ruggeri and M. Sugiyama: submitted to *Cont. Mech. Thermodyn.*